Calculations In Chemistry

A Note to Students

The goal of these lessons is to help you solve *calculations* in first-year chemistry. This is only one part of a course in chemistry, but it is often the most challenging.

Provisions: A *spiral notebook* is suggested as a place to write your work when solving the problems in these lessons. You will also need

- two packs of 100 3x5 index cards (two or more colors are preferred) plus a small assortment of rubber bands, and
- a pack of large (3 to 6 inch long) sticky notes to use as cover sheets.

Choosing a Calculator: As you solve problems, use the *same* calculator that you will be allowed to use during tests, to learn and practice the rules for that calculator before tests.

Many courses will *not* allow the use of a graphing calculator or other calculators with extensive memory during tests. If no type of calculator is specified for your course, any inexpensive calculator with a [1/x] *or* [x^{-1}], [y^x] or [^], [log] *or* [10^x], *and* [ln] functions will be sufficient for most calculations in first-year chemistry.

Buy *two* identical calculators if possible. If one becomes broken or lost, you will have a familiar backup if the bookstore is sold out later in the term.

When to Start: You will receive the maximum benefit from these lessons by completing the lessons on a topic *before* it is addressed in lecture.

Where to Start: The order of these lessons may not always match the order in which topics are covered in your course. If you are using these modules as part of a course, complete the lessons in the order in which they are assigned by your instructor.

If you are using these lessons "on your own" to assist with a course,

- First, determine the *topics* that will be covered on your *next* graded problem set, quiz, or test.
- Find those topics in the Table of Contents.
- Download the modules that include the topics.
- Find the *prerequisite* lessons for the topic, listed at the beginning of the module or lesson. Download and print those lessons. Do the prerequisites, then the topics related to your next graded assignments.
- Follow the instructions on "How to Use These Lessons" on page 1.

If you begin these lessons after the start of your course, as time permits, review prior topics starting with Module 1. You will need all of these introductory modules for later topics -- and for your final exam.

Check back for updates at www.ChemReview.Net .

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How to Use These Lessons

1. ***Read* the lesson. *Work* the questions (Q).** As you read, use this method.

   - As you *turn* to two new facing pages in this book, cover the page on the *right* with a sheet of paper.
   - As you start any new page, *if* you see 5 stars (* * * * *) on the page, ***cover*** the text below the stars. As a cover sheet, use either overlapping sticky notes or a folded sheet of paper.
   - In your problem notebook, write your answer to the question (**Q**) above the * * * * * . Then slide down the cover sheet to the next set of * * * * * and check your answer. If you need a hint, read a part of the answer, then re-cover the answer and try the problem again.

2. **Memorize the rules, *then* do the Practice.**

   The goal in learning is to move rules and concepts into *memory*. To begin, when working questions (**Q**) in a lesson, you may look back at the rules, but make an effort to commit the rules to memory before starting the **Practice** problems.

   Try every *other* problem of a **Practice** set on the first day and the remaining problems in your next study session. This spacing will help you to remember new material. On both days, try to work the **Practice** without looking back at the rules.

   Answers to the **Practice** are the end of each lesson. If you need a hint, read a part of the answer and try again.

3. **How many Practice problems should you do?** It depends on your background. These lessons are intended to

   - refresh your memory on topics you once knew, and
   - fill-in the gaps for topics that are less familiar.

   If you know a topic well, read the lesson for review, then do a *few* problems on each **Practice** set. Be sure to do the last problem (usually the most challenging).

   If a topic is unfamiliar, do more problems.

4. **Work Practice problems at least 3 days a week.** Chemistry is cumulative. To solve problems, what you learn early you will need in memory later. To retain what you learn, *space* your study of a topic over several days.

   Science has found that your memory tends to retain what it uses repeatedly, but to remember for only a few days what you do not practice over several days. If you wait until a quiz deadline to study, what you learn may remain in memory for a day or two, but on later tests and exams, it will tend to be forgotten.

   Begin lessons on new topics early, preferably before the topic is covered in lecture.

5. **Memorize what must be memorized.** Use flashcards and other memory aids.

Chemistry is not easy, but you will achieve success if you work at a *steady* pace.

If you have previously taken a course in chemistry, many topics in Modules 1 to 4 will be review. Therefore: if you can pass the pre-test for a lesson, skip the lesson. If you need a bit of review to refresh your memory, do the last few problems of each **Practice** set. On topics that are less familiar, complete more **Practice**.

Module 1 – Scientific Notation

Timing: Module 1 should be done as soon as you are assigned problems that use exponential notation. If possible, do these lessons *before* attempting problems in other textbooks.

Additional Math Topics

Powers and *roots* of exponential notation are covered in Lesson 28B.

Complex units such as → are covered in Lesson 17C.

$$\frac{\text{atm} \bullet \text{L}}{(\text{mole})\left(\frac{\text{atm} \bullet \text{L}}{\text{mole} \bullet \text{K}}\right)}$$

Those lessons may be done at any time after Module 1.

Calculators and Exponential Notation

To multiply 492 x 7.36, the calculator is a useful tool. However, when using exponential notation, you will make fewer mistakes if you do as much exponential math as you can without a calculator. These lessons will review the rules for doing exponential math "in your head."

The majority of problems in Module 1 will *not* require a calculator. Problems that require a calculator will be clearly identified.

You are encouraged to try complex problems with the calculator *after* you have tried them without. This should help you to decide when, and when not, to use a calculator.

Notation Terminology

In science, we often deal with very large and very small numbers.

For example: A drop of water contains about **1,500,000,000,000,000,000,000** molecules.

An atom of neon has a radius of about **0.000 000 007 0** centimeters.

When values are expressed as "regular numbers," such as 123 or 0.024 or the numbers above, they are said to be in **fixed decimal** or **fixed** notation.

Very large and small numbers are more clearly expressed in **exponential notation:** writing a *number* times **10** to a *whole-number* power. For the measurements above, we can write

- A drop of water contains about $\mathbf{1.5 \times 10^{21}}$ molecules.
- An atom of neon gas has a radius of about $\mathbf{7.0 \times 10^{-9}}$ centimeters.

Values represented in exponential notation can be described as having three parts.

For example, in $\mathbf{-6.5 \times 10^{-4}}$,

- The — in front is the *sign*.
- the ***6.5*** is termed the *significand* or *decimal* or *digit* or *mantissa* or *coefficient*.
- The $\mathbf{10^{-4}}$ is the *exponential* term: the *base* is **10** and the *exponent* (or *power*) is **—4**.

In these lessons we will refer to the two parts of exponential notation after the sign as the **significand** and **exponential** term.

sign ↓
$\mathbf{-\ 6.5\ \times\ 10^{-4}}$
↑ ↑
significand exponential

You should also learn (and use) any alternate terminology preferred in your course.

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Lesson 1A: Moving the Decimal

Pretest: Do *not* use a calculator. If you get a perfect score on this pretest, skip to Lesson 1B. Otherwise, complete Lesson 1A. Answers are at the end of each *lesson*.

1. Write these in *scientific* notation.

   a. $9{,}400 \times 10^3$ = ____________________ b. 0.042×10^6 = __________________

   c. -0.0067×10^{-2} = __________________ d. -77 = __________________

2. Write these answers in fixed decimal notation.

   a. 14/10,000 = b. 0.194×1000 = c. 47^0=

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Powers of 10

Below are the numbers that correspond to powers of 10. Note the relationship between the *exponents* and position of the *decimal point* in the fixed decimal numbers as you go down the sequence.

10^6 = 1,000,000
10^3 = 1,000 = 10 x 10 x 10
10^2 = 100
10^1 = 10
10^0 = **1** (*Anything* to the *zero* power equals *one*.)
10^{-1} = 0.1
10^{-2} = 0.01 = $1/10^2$ = 1/100
10^{-3} = 0.001

When converting from powers of 10 to fixed decimal numbers, use these steps.

1. To change a *positive* power of 10 to a fixed decimal *number*,
   - write **1,** then after 1 add the number of zeros equal to the exponent.

     Example: 10^2 = 1**00** Another way to state this rule:

     ∪∪↑

   - From **1**, move the decimal to the *right* by the number of places in the exponent.
2. To convert a *negative* power of 10 to fixed notation,
   - From 1, move the decimal to the *left* by the number of places equal to the exponent of 10 *after* its negative sign.

     Question: 10^{-2} = (fill in the fixed decimal number.)

* * * * * (See *How To Use These Lessons, Point 1,* on page 1).

Answer: 10^{-2} = **0.01**

↑∪∪

Practice A: Write your answers, then check them at the end of this *lesson*.

1. Write these as regular numbers without an exponential term.

   a. 10^4 = _______________ b. 10^{-4} = _______________

   c. 10^7 = _______________ d. 10^{-5} = _______________ e. 10^0 = ________

Multiplying and Dividing By 10, 100, 1000

To multiply or divide by *numbers* that are positive whole-number powers of 10, such as 100 or 10,000, use these rules.

1. When *multiplying* a number by a 10, 100, 1000, etc., move the decimal to the *right* by the number of *zeros* in the 10, 100, or 1000.

   Examples: 72 x 100 = 7,200 — 0.0624 x 1,000 = — 62.4

2. When *dividing* a number by 10, 100, or 1000, etc., move the decimal to the *left* by the number of *zeros* in the 10, 100, or 1000.

   Q: Answer as fixed decimal numbers: 34.6/1000 = 0.47/100 =

* * * * *

Answers: 34.6/1000 = 0.0346 0.47/100 = 0.0047

↑∪∪∪

3. When writing a number that has a value between —1 and 1 (a number that "begins with a decimal point"), always place a *zero* in *front* of the *decimal* point.

   Example: Do *not* write .42 or — .74 ; *do* write **0**.42 or — **0.**74

   During your written calculations, the zero in front helps in seeing your decimals.

Practice B: *Write* your answers, then check them at the end of this lesson.

1. When dividing by 1,000 move the decimal to the _______________ by _____ places.
2. Write these answers as fixed decimal numbers.

   a. 0.42 x 1000 =   b. 63/100 =   c. $-$74.6/10,000 =

Converting Exponential Notation to Numbers

To convert from exponential notation (such as -4×10^{3}) to fixed decimal notation ($-$4,000), use these rules.

1. The sign in front does not change. The sign is independent of the terms after the sign.
2. If, in the exponential notation, the significand is multiplied by a

- *positive* power of 10, move the decimal point in the significand to the *right* by the same number of places as the value of the exponent;

  Examples: 2×10^{2} = **200** -0.0033×10^{3} = **$-$3.3**

- *negative* power of 10, move the decimal point in the significand to the *left* by the same number of places as the number *after* the minus sign of the exponent.

  Examples: 2×10^{-2} = **0.02** $-7{,}653.8 \times 10^{-3}$ = **$-$7.6538**

Practice C: Convert these to fixed decimal notation.

1. 3×10^{3} = ____________________   2. 5.5×10^{-4} = ________________________

3. 0.77×10^{6} = _________________   4. -95×10^{-4} = _____________________

Changing Exponential to Scientific Notation

In chemistry, it is often required that numbers that are very large or very small be written in **scientific** notation. One reason to use scientific notation is that it makes values easier to compare: there are many equivalent ways to write a value in exponential notation, but only one correct way to write the value in scientific notation.

Scientific notation is simply a special case of exponential notation in which the significand is *1 or greater*, but *less* than *10*, and is multiplied by 10 to a whole-number power.

Another way to say this: in scientific notation, the *decimal point* in the significand must be *after* the first digit that is not a zero.

Example: -0.057×10^{-2} in scientific notation is written as **$-$5.7** $\times 10^{-4}$.

The decimal must be moved to after the first number that is not a zero: the 5.

> To convert from exponential to *scientific* notation,
> - move the decimal in the significand to *after* the first digit that is not a zero, then
> - adjust the exponent to keep the same numeric value.

When moving a decimal point,

1. The *sign* in front does not change.
2. If moving the decimal Y times to make the significand *larger*, make the power of 10 *smaller* by a *count* of Y.

   Example. Converting exponential to *scientific* notation: $0.045 \times 10^5 = 4.5 \times 10^{3}$
   ◡◡↑

   The decimal must be after the 4. Move the decimal two times to the right. This makes the significand 100 times larger. To keep the same numeric value, lower the exponent by 2, making the 10^x value 100 times smaller.

3. When moving the decimal Y times to make the significand *smaller*, make the power of 10 *larger* by a count of Y.

   Q. Convert to scientific notation: $-8{,}544 \times 10^{-7} =$

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   Answer: $-8{,}544 \times 10^{-7} = -8.544 \times 10^{-4}$
   ↑◡◡◡

   You must move the decimal 3 places to the left. This makes the significand 1,000 times smaller. To keep the same numeric value, increase the exponent by 3, making the 10^x value 1,000 times larger.

   Remember, 10^{-4} is 1,000 times *larger* than 10^{-7}.

It helps to *recite*, every time you move a decimal, for the terms after the sign in front:

> "If one gets *smaller*, the other gets *larger*. If one gets larger, the other gets smaller."

Practice D: Change these to scientific notation. Check answers at the end of the lesson.

1. $5{,}420 \times 10^3 =$ ____________________
2. $0.0067 \times 10^{-4} =$ ____________________
3. $0.020 \times 10^3 =$ ____________________
4. $-870 \times 10^{-4} =$ ____________________
5. $0.00492 \times 10^{-12} =$ ________________
6. $-602 \times 10^{21} =$ ____________________

Converting Numbers to Scientific Notation

To convert regular (fixed decimal) numbers to *exponential* notation, use these rules.

- Any number to the zero power equals one.

  $2^0 = \mathbf{1}$. $42^0 = \mathbf{1}$. Exponential notation most often uses $\mathbf{10^0 = 1}$.

- Since any number can be multiplied by one without changing its value, any number can be multiplied by 10^0 without changing its value.

  Example: $42 = 42 \times 1 = 42 \times \mathbf{10^0}$ in exponential notation

  $= \mathbf{4.2} \times 10^1$ in *scientific* notation.

To convert fixed notation to *scientific* notation, the *steps* are

1. Add $\mathbf{x\ 10^0}$ after the number.
2. Apply the rules that convert exponential to scientific notation.
   - Do not change the sign in *front*.
   - Write the decimal after the first digit that is not a zero.
   - Adjust the power of 10 to compensate for moving the decimal.

Try: **Q.** Using the steps above, convert these to scientific notation.

a. 943 b. $-$0.00036

* * * * *

Answers: $\mathbf{943} = 943 \times 10^0 = \mathbf{9.43 \times 10^2}$ in scientific notation.

$\mathbf{-0.00036} = -0.00036 \times 10^0 = \mathbf{-3.6 \times 10^{-4}}$ in scientific notation.

When converting to scientific notation, a positive fixed decimal number that is

- *larger* than *one* have a *positive* exponent (zero and above) in scientific notation;
- *between* zero and one have a *negative* exponent in scientific notation; and
- the number of *places* that the decimal in a number moves is the *number* after the sign in its exponent.

These same rules apply to numbers *after* a negative sign in front. The sign in front is independent of the numbers after it.

Note how these three rules apply to the two answers above.

Note also that in both exponential and scientific notation, whether the sign in front is positive or negative has no relation to the sign of the *exponential* term. The sign in front shows whether a value is positive or negative. The exponential term indicates only the position of the decimal point.

Practice E

1. Which lettered parts in Problem 2 below must have exponentials that are negative when written in scientific notation?
2. Change these to scientific notation.

   a. 6,280 = ______________________ b. 0.0093 = _________________________

   c. 0.741 = ______________________ d. – 1,280,000 = _____________________
3. Complete the problems in the *pretest* at the beginning of this lesson.

Study Summary

In your problem notebook,

- write a list of rules in this lesson that were unfamiliar or you found helpful.
- Condense your wording, number the points, and write and recite the rules until you can write them from memory.

Then complete the problems below.

The Role of Practice

Do as many **Practice** problems as you need to feel "quiz ready."

- If the material in a lesson is relatively easy review, do the *last* problem on each series of similar problems.
- If the lesson is less easy, put a check by (✓) and then work every 2nd or 3rd problem. If you miss one, do some similar problem in the set.
- Save a few problems for your next study session -- and quiz/test review.

During Examples and **Q** problems, you *may* look back at the rules, but practice writing and recalling new rules from memory before starting the **Practice**.

If you use the **Practice** to learn the rules, it will be difficult to find time for all of the problems you will need to do. If you use **Practice** to *apply* rules that are in memory, you will need to solve fewer problems to be "quiz ready."

Practice F

Practice F: Check (✓) and do every *other* letter. If you miss one, do another letter for that set. Save a few parts for your next study session.

1. Write these answers in fixed decimal notation.

   a. 924/10,000 = b. 24.3 x 1000 = c. – 0.024/10 =
2. Convert to scientific notation.

   a. 0.55×10^5 b. 0.0092 x 100 c. 940×10^{-6} d. 0.00032×10^1

3. Write these numbers in scientific notation.

   a. 7,700　　b. 160,000,000　　c. 0.023　　d. 0.00067

ANSWERS (To make answer pages easy to locate, use a sticky note.)

Pretest: 1a. 9.4×10^6 1b. 4.2×10^4 1c. -6.7×10^{-5} 1d. -7.7×10^1

2a. 0.0014 2b. 194 2 c. 1

Practice A: 1a. **10,000** b. **0.0001** c. **10,000,000** d. **0.00001**. e. **1**

Practice B: 1. When dividing by 1,000 , move the decimal to the **left** by **3** places.

2a. 0.42 x 1000 = **420** 2b. 63/100 = **0.63** (must have zero in front) c. — 74.6/10,000 = **— 0.00746**

Practice C: 1. 3,000 2. 0.00055 3. 770,000 4. — 0.0095

Practice D: 1. 5.42×10^6 2. 6.7×10^{-7} 3. 2.0×10^1 4. -8.7×10^{-2} 5. 4.92×10^{-15} 6. -6.02×10^{23}

Practice E: 1. 2b and 2c 2a. 6.28×10^3 2b. 9.3×10^{-3} 2c. 7.41×10^{-1} 2d. -1.28×10^6

Practice F: 1a. 0.0924 1b. 24,300 1c. — 0.0024

2a. 5.5×10^4 2b. 9.2×10^{-1} 2c. 9.4×10^{-4} 2d. 3.2×10^{-3}

3a. 7.7×10^3 3b. 1.6×10^8 3c. 2.3×10^{-2} 3d. 6.7×10^{-4}

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Lesson 1B: Calculations Using Exponential Notation

Pretest: If you can answer all of these three questions correctly, you may skip to Lesson 1C. Otherwise, complete Lesson 1B. Answers are at the end of this lesson.

Do *not* use a calculator. Convert final answers to scientific notation.

1. $(2.0 \times 10^{-4})\ (6.0 \times 10^{23}) =$

2. $\dfrac{10^{23}}{(100)(3.0 \times 10^{-8})} =$

3. $(-6.0 \times 10^{-18}) - (-2.89 \times 10^{-16}) =$

* * * * *

Mental Arithmetic

In chemistry, you must be able to solve simple or estimated calculations without a calculator to speed your work and as a check on your calculator answers. This mental arithmetic is simplified by using exponential notation. In this lesson, we will review the rules for doing exponential calculations "in your head."

Adding and Subtracting Exponential Notation

To add or subtract exponential notation without a calculator, the standard rules of arithmetic can be applied – *if* all of the numbers have the *same exponential* term.

Re-writing numbers to have the same exponential term usually results in values that are not in *scientific* notation. That's OK. During calculations, the rule is to *work* in exponential notation, to allow flexibility with decimal point positions, then to convert to scientific notation at the *final* step.

To add or subtract numbers with exponential terms, you may convert all of the exponential terms to *any* consistent power of 10. However, it usually simplifies the arithmetic if you convert all values to the same exponential as the *largest* of the exponential terms being added or subtracted.

The rule is

To add or subtract exponential notation by hand, make all of the exponents the same.

The steps are

> To add or subtract exponential notation without a calculator,
>
> 1. Re-write each number so that all of the significands are multiplied by the *same* power of 10. Converting to the *highest* power of 10 being added or subtracted is suggested.
> 2. Write the significands and exponentials in columns: numbers under numbers (lining up the decimal points), **x** under **x**, exponentials under exponentials.
> 3. Add or subtract the significands using standard arithmetic, then attach the *common* power of 10 to the answer.
> 4. Convert the final answer to scientific notation.

Follow how the steps are applied in this

Example: $(40.71 \times 10^{8}) + (222 \times 10^{6}) = (40.71 \times 10^{8}) + (2.22 \times 10^{8}) =$

$$\begin{array}{rr} & 40.71 \times 10^{8} \\ + & \underline{2.22 \times 10^{8}} \\ & 42.93 \times 10^{8} \end{array} = \boxed{\mathbf{4.293 \times 10^{9}}}$$

Using the steps and the method shown in the example, try the following problem with*out* a calculator. In this problem, do not round numbers during or after the calculation.

Q. $(32.464 \times 10^{1}) - (16.2 \times 10^{-1}) = ?$

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A. $(32.464 \times 10^{1}) - (16.2 \times 10^{-1}) = (32.464 \times 10^{1}) - (\mathbf{0.162} \times 10^{+1}) =$

$$\begin{array}{rr} & 32.464 \ \mathbf{x}\ \mathbf{10^{1}} \\ - & \underline{0.162 \ \mathbf{x}\ \mathbf{10^{1}}} \\ & 32.302 \ \mathbf{x}\ \mathbf{10^{1}} \end{array} = \mathbf{3.2302 \times 10^{2}}$$

(10^{1} has a higher value than 10^{-1})

Let's do problem 1 again. This time, below, convert the exponential notation to *regular numbers,* do the arithmetic, then convert the final answer to scientific notation.

$$\begin{array}{rl} 32.464 \times 10^{1} & = \\ -\ \underline{16.2 \quad \times 10^{-1}} & = \end{array}$$

* * * * *

$$\begin{array}{rlrl} 32.464 \times 10^{1} & = & \mathbf{324.64} & \\ -\ \underline{16.2 \quad \times 10^{-1}} & = & -\ \underline{\mathbf{1.62}} & \\ & & \mathbf{323.02} & = \mathbf{3.2302 \times 10^{2}} \end{array}$$

The answer is the same either way, as it must be. This "convert to regular numbers" method is an option when the exponents are close to 0. However, for exponents such as $10^{\mathbf{23}}$ or $10^{\mathbf{-17}}$, it is easier to use the method above that includes the exponential, but adjusts so that all of the exponentials are the same.

Practice A: Try these with*out* a calculator. On these, don't round. Do convert final answers to scientific notation. Do the odds first, then the evens if you need more practice.

1. $$\begin{array}{rl} & 64.202 \times 10^{23} \\ + & \underline{13.2 \quad \times 10^{21}} \end{array}$$

2. $(61 \times 10^{-7}) + (2.25 \times 10^{-5}) + (212.0 \times 10^{-6}) =$

3. $(-54 \times 10^{-20}) + (-2.18 \times 10^{-18}) =$

4. $(-21.46 \times 10^{-17}) - (-3{,}250 \times 10^{-19}) =$

Multiplying and Dividing Powers of 10

The following boxed rules should be recited until they can be recalled from memory.

1. When you *multiply* exponentials, you *add* the exponents.

   Examples: $10^3 \times 10^2 = 10^{\mathbf{5}}$ $\quad 10^{-5} \times 10^{-2} = 10^{\mathbf{-7}}$ $\quad 10^{-3} \times 10^{5} = 10^{\mathbf{2}}$

2. When you *divide* exponentials, you *subtract* the exponents.

   Examples: $10^3/10^2 = 10^{\mathbf{1}}$ $\quad 10^{-5}/10^{2} = 10^{\mathbf{-7}}$ $\quad 10^{-5}/10^{-2} = 10^{\mathbf{-3}}$

   When subtracting, remember: Minus a minus is a plus. $10^{6-(-3)} = 10^{6+3} = 10^{9}$

3. When you take the reciprocal of an exponential, change the sign.

This rule is often remembered as:

When you take an exponential term from the bottom to the top, change its sign.

Example: $\frac{1}{\mathbf{10^3}}$ = $\mathbf{10^{-3}}$; $1/\mathbf{10^{-5}}$ = $\mathbf{10^5}$

Why does this work? Rule 2: $\frac{1}{\mathbf{10^3}} = \frac{\mathbf{10^0}}{\mathbf{10^3}} = \mathbf{10^{0-3}} = \mathbf{10^{-3}}$

4. When fractions include several terms, it may help to simplify the top and bottom *separately*, then divide.

   Example: $\frac{10^{-3}}{10^5 \times 10^{-2}} = \frac{10^{-3}}{\mathbf{10^3}} = \mathbf{10^{-6}}$

Try the following problem.

Q. Without using a calculator, simplify the top, then the bottom, then divide.

$\frac{10^{-3} \times 10^{-4}}{10^5 \times 10^{-8}}$ = ________ =

* * * * *

Answer: $\frac{10^{-3} \times 10^{-4}}{10^5 \times 10^{-8}} = \frac{\mathbf{10^{-7}}}{\mathbf{10^{-3}}} = \mathbf{10^{-7-(-3)}} = \mathbf{10^{-7+3}} = \mathbf{10^{-4}}$

Practice B: Write answers as 10 to a power. Do *not* use a calculator. Do the odds first, then the evens if you need more practice.

1. $10^6 \times 10^2$ =
2. $10^{-5} \times 10^{-6}$ =
3. $\frac{10^{-5}}{10^{-4}}$ =
4. $\frac{10^{-3}}{10^5}$ =
5. $\frac{1}{10^{-4}}$ =
6. $1/10^{23}$ =
7. $\frac{10^3 \times 10^{-5}}{10^{-2} \times 10^{-4}}$ =
8. $\frac{10^5 \times 10^{23}}{10^{-1} \times 10^{-6}}$ =
9. $\frac{100 \times 10^{-2}}{1{,}000 \times 10^6}$ =
10. $\frac{10^{-3} \times 10^{23}}{10 \times 1{,}000}$ =

Multiplying and Dividing in Exponential Notation

These are the rules we use most often.

1. When multiplying and dividing using exponential notation, handle the *significands* and *exponents separately***.**

> Do number math using number rules, and exponential math using exponential rules. Then combine the two parts.

Apply rule 1 to the following three problems.

a. Do not use a calculator: $(2 \times 10^{3})\ (4 \times 10^{23})$ =

* * * * *

For numbers, use number rules. 2 times 4 is **8**

For exponentials, use exponential rules. $10^{3} \times 10^{23} = 10^{\mathbf{3+23}} = 10^{\mathbf{26}}$

Then combine the two parts: $(2 \times 10^{3})\ (4 \times 10^{23}) = \mathbf{8 \times 10^{26}}$

b. Do the *significand* math on a calculator but try the exponential math in your head for $(2.4 \times 10^{-3})\ (3.5 \times 10^{23})$ =

* * * * *

Handle significands and exponents separately.

- Use a calculator for the numbers. 2.4 x 3.5 = **8.4**
- Do the exponentials in your head. $10^{\mathbf{-3}} \times 10^{\mathbf{23}} = 10^{\mathbf{20}}$
- Then combine.

$(2.4 \times 10^{-3})\ (3.5 \times 10^{23}) = (2.4 \times 3.5) \times (10^{-3} \times 10^{23}) = \mathbf{8.4 \times 10^{20}}$

We will review how much to round answers in Module 3. Until then, round numbers and significands in your answers to *two* digits unless otherwise noted.

c. Do significand math on a calculator but exponential math with*out* a calculator.

$$\frac{6.5 \times 10^{23}}{4.1 \times 10^{-8}} =$$

* * * * *

Answer: $\frac{6.5 \times 10^{23}}{4.1 \times 10^{-8}} = \frac{6.5}{4.1} \times \frac{10^{23}}{10^{-8}} = 1.585 \times [10^{23-(-8)}] = \mathbf{1.6 \times 10^{31}}$

2. When dividing, if an exponential term does *not* have a significand, add a **1 x** in front of the exponential so that the number-number division is clear.

Apply Rule 2 to the following problem. Do *not* use a calculator.

$$\frac{10^{-14}}{2.0 \times 10^{-8}} =$$

* * * * *

Answer: $\frac{10^{-14}}{2.0 \times 10^{-8}} = \frac{1}{2.0} \times \frac{10^{-14}}{10^{-8}} = 0.50 \times 10^{-6} = \mathbf{5.0 \times 10^{-7}}$

Practice C

Study the two rules above, then apply them from memory to these problems. To have room for careful work, solve these in your notebook.

Do the odds first, then the evens if you need more practice. Try these *first* without a calculator, then check your mental arithmetic with a calculator if needed. Write final answers in scientific notation, rounding significands to two digits.

1. $(2.0 \times 10^{1})\ (6.0 \times 10^{23}) =$
2. $(5.0 \times 10^{-3})\ (1.5 \times 10^{15}) =$
3. $\frac{3.0 \times 10^{-21}}{-2.0 \times 10^{3}} =$
4. $\frac{6.0 \times 10^{-23}}{2.0 \times 10^{-4}} =$
5. $\frac{10^{-14}}{-5.0 \times 10^{-3}} =$
6. $\frac{10^{14}}{4.0 \times 10^{-4}} =$
7. Complete the three problems in the *pretest* at the beginning of this lesson.

Study Summary

In your problem notebook, write a list of rules in Lesson 1B that were unfamiliar, need reinforcement, or you found helpful. Then condense your list. Add this new list to your numbered points from Lesson 1A. Write and recite the combined list until you can write all of the points from memory. *Then* do the problems below.

Practice D

Start by doing every *other* letter. If you get those right, go to the next number. If not, do a few more of that number. Save one part of each question for your next study session.

1. Try these with*out* a calculator. Convert final answers to scientific notation.
   a. $3 \times (6.0 \times 10^{23}) =$
   b. $1/2 \times (6.0 \times 10^{23}) =$
   c. $0.70 \times (6.0 \times 10^{23}) =$
   d. $10^{3} \times (6.0 \times 10^{23}) =$
   e. $10^{-2} \times (6.0 \times 10^{23}) =$
   f. $(-0.5 \times 10^{-2})(6.0 \times 10^{23}) =$
   g. $\frac{1}{10^{12}} =$
   h. $1/10^{-9} =$
   i. $\frac{3.0 \times 10^{24}}{6.0 \times 10^{23}} =$
   j. $\frac{2.0 \times 10^{18}}{6.0 \times 10^{23}} =$

k. $\frac{1.0 \times 10^{-14}}{4.0 \times 10^{-5}}$ =

l. $\frac{10^{10}}{2.0 \times 10^{-5}}$ =

2. Use a calculator for the numbers, but not for the exponents.

a. $\frac{2.46 \times 10^{19}}{6.0 \times 10^{23}}$ =

b. $\frac{10^{-14}}{0.0072}$ =

3. Do not use a calculator. Write answers as a power of 10.

a. $\frac{10^{7} \times 10^{-2}}{10 \times 10^{-5}}$ =

b. $\frac{10^{-23} \times 10^{-5}}{10^{-5} \times 100}$ =

4. Convert to scientific notation in the final answer. Do not round during these.

a. $(74 \times 10^{5}) + (4.09 \times 10^{7})$ =

b. $(5.122 \times 10^{-9}) - (-12{,}914 \times 10^{-12})$ =

ANSWERS

Pretest. In scientific notation: 1. 1.2×10^{20} 2. 3.3×10^{28} 3. 2.83×10^{-16}

Practice A: You may do the arithmetic in any way you choose that results in these final answers.

1.
$$\begin{array}{r} 64.202 \times 10^{23} \\ +\ 13.2 \times 10^{21} \\ \hline \end{array} = \begin{array}{r} 64.202 \times 10^{23} \\ +\ 0.132 \times \mathbf{10^{23}} \\ \hline 64.334 \times \mathbf{10^{23}} \end{array} = \mathbf{6.4334 \times 10^{24}}$$

2. $(61 \times 10^{-7}) + (2.25 \times 10^{-5}) + (212.0 \times 10^{-6}) = (0.61 \times 10^{\mathbf{-5}}) + (2.25 \times 10^{-5}) + (21.20 \times 10^{\mathbf{-5}})$ =

$$\begin{array}{r} 0.61 \times \mathbf{10^{-5}} \\ 2.25 \times \mathbf{10^{-5}} \\ +\ 21.20 \times \mathbf{10^{-5}} \\ \hline 24.06 \times \mathbf{10^{-5}} \end{array} = \mathbf{2.406 \times 10^{-4}}$$

(10^{-5} is the *highest* value of the three exponentials)

3. $(-54 \times 10^{-20}) + (-2.18 \times 10^{-18}) = (-\mathbf{0.54} \times \mathbf{10^{-18}}) + (-2.18 \times 10^{-18})$ =

$$\begin{array}{r} -\ 0.54 \times \mathbf{10^{-18}} \\ -\ 2.18 \times 10^{-18} \\ \hline \mathbf{-\ 2.72 \times 10^{-18}} \end{array}$$

(10^{-18} is *higher* in value than 10^{-20})

4. $(-21.46 \times 10^{-17}) - (-3{,}250 \times 10^{-19})$ =

$(\mathbf{+}\,3{,}250 \times 10^{-19}) - (21.46 \times 10^{-17}) = (\mathbf{+}\,32.50 \times 10^{\mathbf{-17}}) - (21.46 \times 10^{-17})$ =

$$\begin{array}{r} 32.50 \times 10^{\mathbf{-17}} \\ -\ 21.46 \times 10^{-17} \\ \hline \mathbf{11.04 \times 10^{-17}} \end{array} = \mathbf{1.104 \times 10^{-16}}$$

Practice B

1. 10^8 2. 10^{-11} 3. 10^{-1} 4. 10^{-8} 5. 10^4 6. 10^{-23} 7. 10^4 8. 10^{35}

9. $\frac{100 \times 10^{-2}}{1{,}000 \times 10^6} = \frac{10^2 \times 10^{-2}}{10^3 \times 10^6} = \frac{10^0}{10^9} = \mathbf{10^{-9}}$

10. $\frac{10^{-3} \times 10^{23}}{10 \times 1{,}000} = \frac{10^{20}}{10^4} = \mathbf{10^{16}}$

(For 9 and 10, you may use different steps, but you must arrive at the same answer.)

Practice C

1. 1.2×10^{25} 2. 7.5×10^{12} 3. -1.5×10^{-24} 4. 3.0×10^{-19} 5. -2.0×10^{-12} 6. 2.5×10^{17}

Practice D

1a. $3 \times (6.0 \times 10^{23}) = 18 \times 10^{23} = \mathbf{1.8 \times 10^{24}}$

1b. $1/2 \times (6.0 \times 10^{23}) = \mathbf{3.0 \times 10^{23}}$

1c. $0.70 \times (6.0 \times 10^{23}) = \mathbf{4.2 \times 10^{23}}$

1d. $10^3 \times (6.0 \times 10^{23}) = \mathbf{6.0 \times 10^{26}}$

1e. $10^{-2} \times (6.0 \times 10^{23}) = \mathbf{6.0 \times 10^{21}}$

1f. $(-0.5 \times 10^{-2})(6.0 \times 10^{23}) = \mathbf{-3.0 \times 10^{21}}$

1g. $\frac{1}{10^{12}} = \mathbf{10^{-12}}$

1h. $1/10^{-9} = \mathbf{10^9}$

1i. $\frac{3.0 \times 10^{24}}{6.0 \times 10^{23}} = \frac{3.0}{6.0} \times \frac{10^{24}}{10^{23}} = 0.50 \times 10^1 = \mathbf{5.0 \times 10^0}$ (= **5.0**)

1j. $\frac{2.0 \times 10^{18}}{6.0 \times 10^{23}} = 0.33 \times 10^{-5} = \mathbf{3.3 \times 10^{-6}}$

1k. $\frac{1.0 \times 10^{-14}}{4.0 \times 10^{-5}} = 0.25 \times 10^{-9} = \mathbf{2.5 \times 10^{-10}}$

1l. $\frac{10^{10}}{2.0 \times 10^{-5}} = \frac{1}{2.0} \times \frac{10^{10}}{10^{-5}} = 0.50 \times 10^{15} = \mathbf{5.0 \times 10^{14}}$

2a. $\frac{2.46 \times 10^{19}}{6.0 \times 10^{23}} = 0.41 \times 10^{-4} = \mathbf{4.1 \times 10^{-5}}$

2b. $\frac{10^{-14}}{0.0072} = \frac{1.0 \times 10^{-14}}{7.2 \times 10^{-3}} = \frac{1.0}{7.2} \times \frac{10^{-14}}{10^{-3}} = 0.14 \times 10^{-11} = \mathbf{1.4 \times 10^{-12}}$

3a. $\frac{10^7 \times 10^{-2}}{\mathbf{10^1} \times 10^{-5}} = \frac{10^5}{10^{-4}} = \mathbf{10^9}$

3b. $\frac{10^{-23} \times \cancel{10^{-5}}}{\cancel{10^{-5}} \times \mathbf{10^2}} = \mathbf{10^{-25}}$

4a. $(74 \times 10^5) + (4.09 \times 10^7) =$

$= (\mathbf{0}.74 \times 10^{\mathbf{7}}) + (4.09 \times 10^7) =$

$$\begin{array}{r} \mathbf{0.74 \times 10^7} \\ + 4.09 \times 10^7 \\ \hline \mathbf{4.83 \times 10^7} \end{array}$$

4b. $(5.122 \times 10^{-9}) - (-12{,}914 \times 10^{-12}) =$

$= (5.122 \times 10^{-9}) \mathbf{+} (12.914 \times 10^{\mathbf{-9}}) =$

$$\begin{array}{r} 5.122 \times 10^{-9} \\ + 12.914 \times 10^{-9} \\ \hline 18.036 \times 10^{-9} \end{array} = \mathbf{1.8036 \times 10^{-8}}$$

* * * * *

Lesson 1C: Tips for Exponential Calculations

Pretest: If you can solve both problems correctly, skip this lesson. Convert your final answers to scientific notation. Check your answers at the end of this lesson.

1. Solve this problem with*out* a calculator. $\frac{(10^{-9})(10^{15})}{(4 \times 10^{-4})(2 \times 10^{-2})} =$

2. For this problem, use a calculator as needed. $\frac{(3.15 \times 10^{3})(4.0 \times 10^{-24})}{(2.6 \times 10^{-2})(5.5 \times 10^{-5})} =$

* * * * *

Choosing a Calculator

If you have not already done so, please read *Choosing a Calculator* under *Notes to the Student* in the preface to these lessons.

Complex Calculations

The prior lessons covered the fundamental rules for exponential notation. For longer calculations, the rules are the same. The challenges are keeping track of the numbers *and* using the calculator correctly. The steps below will help you to simplify complex calculations, minimize data-entry mistakes, and quickly *check* your answers.

Let's try the following calculation two ways.

$$\frac{(7.4 \times 10^{-2})(6.02 \times 10^{23})}{(2.6 \times 10^{3})(5.5 \times 10^{-5})} =$$

Method 1. Do numbers and exponents separately.

Work the calculation above using the following steps.

a. **Do the numbers on the calculator.** Ignoring the exponentials, use the calculator to multiply all of the *significands* on top. Write the result. Then multiply all the significands on the bottom and write the result. Divide, write your answer rounded to two digits, and then check below.

* * * * * (See *How To Use These Lessons, Point 1,* on page 1).

$$\frac{7.4 \times 6.02}{2.6 \times 5.5} = \frac{44.55}{14.3} = \boxed{\mathbf{3.1}}$$

b. **Then exponents.** Starting from the original problem, look only at the powers of 10. Try to solve the exponential math "in your head" with*out* the calculator. Write the answer for the top, then the bottom, then divide.

* * * * *

$$\frac{10^{-2} \times 10^{23}}{10^{3} \times 10^{-5}} = \frac{10^{21}}{10^{-2}} = 10^{21-(-2)} = \boxed{\mathbf{10^{23}}}$$

c. **Now combine** the significand and exponential and write the final answer.

* * * * *

3.1×10^{23}

Note that by grouping the numbers and exponents separately, you did not need to enter the exponents into your calculator. To multiply and divide powers of 10, you simply add and subtract whole numbers.

Let's try the calculation a second way.

Method 2. All on the calculator.

Enter *all* of the numbers and exponents into your calculator. (Your calculator manual, which is usually available online, can help.) Write your final answer in scientific notation. Round the significand to two digits.

* * * * *

On most calculators, you will need to use an E *or* EE *or* EXP or ^ key, rather than the *times* key, to enter a "10 to a power" term.

If you needed that hint, try again, and then check below.

* * * * *

Note how your calculator *displays* the *exponential* term in answers. The exponent may be set apart at the right, sometimes with an **E** in front.

Your calculator answer, rounded, should be the same as with Method 1: $\mathbf{3.1} \times 10^{\mathbf{23}}$.

Which way was easier? "Numbers, then exponents," or "all on the calculator?" How you do the arithmetic is up to you, but "numbers, then exponents" is often quicker and easier.

Checking Calculator Results

Whenever a complex calculation is done on a calculator, you *must* do the calculation a *second* time, using different steps, to catch errors in calculator use. Calculator results can be checked either by using a different key sequence *or* by estimating answers.

"Mental arithmetic estimation" is often the fastest way to check a calculator answer. To learn this method, let's use the calculation that was done in the first section of this lesson.

$$\frac{(7.4 \times 10^{-2})(6.02 \times 10^{23})}{(2.6 \times 10^{3})(5.5 \times 10^{-5})} =$$

Apply the following steps to the problem above.

1. ***Estimate* the numbers first.** Ignoring the exponentials, *round* and then multiply all of the top significands, and write the result. *Round* and multiply the bottom significands. Write the result. Then write a *rounded estimate* of the answer when you divide those two numbers, and then check below.

* * * * *

Your rounding might be

$$\frac{7 \times \cancel{6}}{3 \times \cancel{6}} = \frac{7}{3} \approx \mathbf{2}$$ (the ≈ sign means *approximately equals*)

If your mental arithmetic is good, you can estimate the number math on the paper without a calculator. The estimate needs to be fast, but does *not* need to be exact. If needed, evaluate the *rounded* top and bottom numbers on the calculator, but *try* to practice the arithmetic "in your head."

2. **Simplify the exponents.** The exponents can be solved by adding and subtracting whole numbers. Try the exponential math without the calculator.

   * * * * *

$$\frac{10^{-2} \times 10^{23}}{10^{3} \times 10^{-5}} = \frac{10^{21}}{10^{-2}} = 10^{21-(-2)} = \mathbf{10^{23}}$$

3. **Combine** the estimated number and exponential answers. Compare this estimated answer to answer found when you did this calculation in the section above using a calculator. Are they close?

   * * * * *

   The estimate is 2×10^{23}. The answer with the calculator was 3.1×10^{23}. Allowing for rounding, the two results are close.

   If your fast, rounded, "done in your head" answer is *close* to the calculator answer, it is likely that the calculator answer is correct. If the two answers are *far* apart, check your work.

Estimating Number Division

If you know your multiplication tables, and if you memorize these simple **decimal equivalents** to help in estimating division, you may be able to do many numeric estimates without a calculator.

1/2 = 0.50	1/3 = 0.33	1/4 = 0.25	1/5 = 0.20	2/3 = 0.67	3/4 = 0.75	1/8 = 0.125

The method used to get your *final* answer should be slow and careful. Your *checking* method should use different steps or calculator keys, or, if time is a factor, should use rounded numbers and quick mental arithmetic.

On timed tests, you may want to do the exact calculation first, and then go back at the end, if time is available, and use rounded numbers as a check. When doing a calculation the second time, try not to look back at the first answer until *after* you write the estimate. If you look back, by the power of suggestion, you will often arrive the first answer whether it is correct or not.

For complex operations on a calculator, work each calculation a *second* time using rounded numbers and/or different steps or keys.

Practice

For problems 1-4, you will need to know the "fraction to decimal equivalent" conversions in the box above. If you need practice, try this.

- On a sheet of paper, draw 5 columns and 7 rows. List the fractions down the middle column.
- Write the decimal equivalents of the fractions at the far right.
- Fold over those answers and repeat at the far left. Fold over those and repeat.

		1/2		
		1/3		
		1/4		
		…		

To start, complete the even numbered problems. If you get those right, go to the next lesson. If you need more practice, do the odds.

Then try these next four with*out* a calculator. Convert final answer to scientific notation.

1. $\dfrac{4 \times 10^{3}}{(2.00)(3.0 \times 10^{7})} =$

2. $\dfrac{1}{(4.0 \times 10^{9})(2.0 \times 10^{3})} =$

3. $\dfrac{(3 \times 10^{-3})(8.0 \times 10^{-5})}{(6.0 \times 10^{11})(2.0 \times 10^{-3})} =$

4. $\dfrac{(3 \times 10^{-3})(3.0 \times 10^{-2})}{(9.0 \times 10^{-6})(2.0 \times 10^{1})} =$

For problems 5-8 below, in your notebook

- First write an *estimate* based on rounded numbers, then exponentials. *Try* to do this estimate without using a calculator.
- Then calculate a more precise answer. You may
  - o plug the entire calculation into the calculator, or
  - o use the "numbers on calculator, exponents on paper" method, or
  - o experiment with both approaches to see which is best for you.

Convert both the estimate and the final answer to *scientific notation*. Round the significand in the answer to two digits. Use the calculator that you will be allowed to use on quizzes and tests.

5. $\dfrac{(3.62 \times 10^{4})(6.3 \times 10^{-10})}{(4.2 \times 10^{-4})(9.8 \times 10^{-5})} =$

6. $\dfrac{10^{-2}}{(750)(2.8 \times 10^{-15})} =$

7. $\dfrac{(1.6 \times 10^{-3})(4.49 \times 10^{-5})}{(2.1 \times 10^{3})(8.2 \times 10^{6})} =$

8. $\dfrac{1}{(4.9 \times 10^{-2})(7.2 \times 10^{-5})} =$

9. For additional practice, do the two *pretest* problems at the beginning of this lesson.

ANSWERS

Pretest: 1. 1.25×10^{11} 2. 8.8×10^{-15}

Practice: You may do the arithmetic using different steps than below, but you must get the same *answer*.

1. $\frac{4 \times 10^3}{(2.00)(3.0 \times 10^7)} = \frac{4}{6} \times 10^{3-7} = \frac{2}{3} \times 10^{-4} = 0.67 \times 10^{-4} = \mathbf{6.7 \times 10^{-5}}$

2. $\frac{1}{(4.0 \times 10^9)(2.0 \times 10^3)} = \frac{1}{8 \times 10^{12}} = \frac{1}{8} \times 10^{-12} = 0.125 \times 10^{-12} = \mathbf{1.25 \times 10^{-13}}$

3. $\frac{(\cancel{3} \times 10^{-3})(8.0 \times 10^{-5})}{(\mathbf{2}\ \cancel{6.0} \times 10^{11})(2.0 \times 10^{-3})} = \frac{8}{4} \times \frac{10^{-3-5}}{10^{11-3}} = 2 \times \frac{10^{-8}}{10^8} = 2 \times 10^{-8-8} = \mathbf{2.0 \times 10^{-16}}$

4. $\frac{(3 \times 10^{-3})(3.0 \times 10^{-2})}{(9.0 \times 10^{-6})(2.0 \times 10^1)} = \frac{9}{18} \times \frac{10^{-3-2}}{10^{-6+1}} = 0.50 \times \frac{\cancel{10^{-5}}}{\cancel{10^{-5}}} = 0.50 = \mathbf{5.0 \times 10^{-1}}$

5. First the estimate. The rounding for the *numbers* might be

   $\frac{\cancel{4} \times 6}{\cancel{4} \times 10} = \mathbf{0.6}$ For the *exponents*: $\frac{10^4 \times 10^{-10}}{10^{-4} \times 10^{-5}} = \frac{10^{-6}}{10^{-9}} = 10^9 \times 10^{-6} = \mathbf{10^3}$

   $\approx 0.6 \times 10^3 \approx \mathbf{6 \times 10^2}$ (*estimate*) in scientific notation.

   For the *precise* answer, doing numbers and exponents separately,

   $\frac{(3.62 \times 10^4)(6.3 \times 10^{-10})}{(4.2 \times 10^{-4})(9.8 \times 10^{-5})} = \frac{3.62 \times 6.3}{4.2 \times 9.8} = \boxed{0.55}$ The exponents are done as in the estimate above.

   $= 0.55 \times 10^3 = \boxed{\mathbf{5.5 \times 10^2}}$ (final) in scientific notation.

   This is close to the estimate, a check that the more precise answer is correct.

6. Estimate: $\frac{1}{7 \times 3} \approx \frac{1}{20} = \mathbf{0.05}$; $\frac{10^{-2}}{(10^2)(10^{-15})} = 10^{-2-(-13)} = \mathbf{10^{11}}$

   $0.05 \times 10^{11} = \mathbf{5 \times 10^9}$ (estimate)

   Numbers on calculator: $\frac{1}{7.5 \times 2.8} = \boxed{0.048}$ Exponents – same as in estimate.

   FINAL: $\mathbf{0.048} \times 10^{11} = \boxed{\mathbf{4.8 \times 10^9}}$ (close to the estimate)

7. You might estimate, for the numbers first,

   $\frac{1.6 \times 4.49}{2.1 \times 8.2} = \frac{2 \times 4}{2 \times 8} = \mathbf{0.5}$ For the exponents: $\frac{10^{-3} \times 10^{-5}}{10^3 \times 10^6} = \frac{10^{-8}}{10^9} = \mathbf{10^{-17}}$

   $= 0.5 \times 10^{-17} = \mathbf{5 \times 10^{-18}}$ (estimate)

More precisely, using numbers then exponents, with numbers on the calculator,

$\frac{1.6 \times 4.49}{2.1 \times 8.2} = \boxed{\mathbf{0.42}}$ The exponents are done as in the estimate above.

$0.42 \times 10^{-17} = \boxed{\mathbf{4.2 \times 10^{-18}}}$ This is close to the estimate. Check!

8. Estimate: $\frac{1}{5 \times 7} \approx \frac{1}{35} \approx \mathbf{0.03}$; $\frac{1}{(10^{-2})(10^{-5})} = 1/(10^{-7}) = \mathbf{10^7}$

$0.03 \times 10^7 \approx \mathbf{3 \times 10^5}$ (estimate)

Numbers on calculator $= \frac{1}{4.9 \times 7.2} = \boxed{0.028}$ Exponents – see estimate.

FINAL: $0.028 \times 10^7 = \boxed{\mathbf{2.8 \times 10^5}}$ (close to the estimate)

* * * * *

Lesson 1D: Special Project – The Atoms (Part 1)

At the center of chemistry are **atoms**: the building blocks of matter. There are 91 different kinds of atoms found in the earth's crust. When a substance is stable at room temperature and pressure and contains only one kind of atom, the substance is said to be an **element**, and the atoms are in their **elemental** form.

The Periodic Table helps in predicting the properties of the elements and atoms. In first-year chemistry, about 40 of the atoms are frequently encountered. Quick, automatic conversion between of the names and symbols of those atoms "in your head" will speed and simplify solving problems.

To begin to learn those atoms, **your assignment is:**

- For the **12** atoms below, memorize the name, and symbol, and the position of the atom in the table.
- For each atom, given either its symbol or name, be able to write the other.
- Be able to fill in an empty chart like the one below with these atom names and symbols.

Periodic Table

1A	2A		3A	4A	5A	6A	7A	8A
1 H Hydrogen								2 He Helium
3 Li Lithium	4 Be Beryllium		5 B Boron	6 C Carbon	7 N Nitrogen	8 O Oxygen	9 F Fluorine	10 Ne Neon
11 Na Sodium	12 Mg Magnesium							

* * * * *

SUMMARY – Scientific Notation

1. When writing a number between -1 and 1, place a *zero* in *front* of the *decimal* point. Do *not* write $.42$ or $-.74$; *do* write $\mathbf{0}.42$ or $-\mathbf{0}.74$
2. *Exponential* notation represents numeric values in three parts:
   - a *sign* in front showing whether the value is positive or negative;
   - a *number* (the significand);
   - times a *base* taken to a *power* (the exponential term).
3. In *scientific* notation, the significand must be a number that is 1 or greater, but less than 10, and the exponential term must be 10 to a whole-number power. This places the decimal point in the significand after the first number which is not a zero.
4. When moving a decimal in exponential notation, the sign in front never changes.
5. To keep the same numeric value when moving the decimal of a number in base 10 exponential notation, if you
   - move the decimal *Y times* to make the significand *larger*, make the exponent *smaller* by a *count* of Y;
   - move the decimal Y times to make the significand smaller, make the exponent larger by a count of Y.

   Recite and repeat to remember: When moving the decimal, for the numbers after the sign in front,

"If one gets *smaller*, the other gets *larger*. If one gets *larger*, the other gets *smaller*."

6. To *add or subtract* exponential notation by hand, all of the values must be converted to have the same exponential term.
   - Convert all of the values to have the same power of 10.
   - List the significands and exponential in columns.
   - Add or subtract the significands.
   - Attach the common exponential term to the answer.
7. In *multiplication and division* using scientific or exponential notation, handle numbers and exponential terms separately. Recite and repeat to remember:
   - Do numbers by number rules and exponents by exponential rules.
   - When you multiply exponentials, you add the exponents.
   - When you divide exponentials, you subtract the exponents.
   - When you take an exponential term to a power, you multiply the exponents.
   - To take the reciprocal of an exponential, change the sign of the exponent.
8. In calculations using exponential notation, try the significands on the calculator but the exponents on paper.
9. For complex operations on a calculator, do each calculation a *second* time using rounded numbers and/or different steps or keys.

# # # #

Module 2 – The Metric System

Lesson 2A: Metric Fundamentals

Have you previously mastered the metric system? If you get a perfect score on the following *pretest*, you may skip to Lesson 2B. If not, complete Lesson 2A.

Pretest: Write answers to these, then check your answers at the end of Lesson 2A.

1. What is the mass, in kilograms, of 150 cc's of liquid water?
2. How many cm^3 are in a liter? How many dm^3 are in a liter?
3. 2.5 pascals is how many millipascals?
4. 3,500 centigrams is how many kilograms?

* * * * *

The Importance of Units

The fastest and most effective way to solve problems in chemistry is to focus on the **units** that measure quantities. In science, measurements and calculations are done using the **metric system**.

All measurement systems begin with **standards** *defining* **fundamental quantities** that include **distance**, **mass**, and **time**.

Distance

The metric distance unit is the **meter,** abbreviated **m**. One meter is about 39.3 inches, slightly longer than one yard. A meter stick is usually numbered in **centimeters.**

```
|||||||||10|||||||||20|||||||||30|||||||||40|||||||||50|||||||||60|||||||||70|||||||||80|||||||||90|||||||||
```

Just as a dollar can be divided into 100 *cent*s, and a *cent*ury is 100 years, a meter is divided into 100 *centi*meters. The centimeter, abbreviated **cm**, is 1/100th of a meter. As equalities, we can write

1 **centi**meter ≡ 1/100th of a meter ≡ 10^{-2} meters *and* 1 meter ≡ **100** *centi*meters

The symbol ≡ means "is *defined* as equal to" and/or "is *exactly* equal to."

A centimeter can be divided into 10 **milli**meters (**mm**). Each millimeter is 1/1000th of a meter. In relation to the meter,

1 **milli**meter ≡ 1/1,000th of a meter ≡ 10^{-3} meters *and* 1 meter ≡ **1,000** *milli*meters

A meter stick can also be divided into 10 **deci**meters (**dm**). As equalities,

1 **deci**meter ≡ 1/10th of a meter ≡ 10^{-1} meters *and* 1 meter ≡ **10** *deci*meters

One decimeter is also equal to 10 centimeters.

Long distances are usually measured in **kilo**meters (**km**). **1 kilometer ≡ 1,000 meters**

What do you need to remember from the above? You will need to be able to write from memory the following two rules.

1. The "meter-stick" equalities

1 METER ≡ 10 **deci**METERS ≡ 100 **centi**METERS ≡ 1,000 **milli**METERS

and 1,000 METER sticks ≡ 1 **kilo**METER

2. The "one prefix" definitions

1 **milli**METER ≡ 1 **mm** ≡ 10^{-3} METERS (≡ 1/1000th METER ≡ 0.001 METERS)

1 **centi**METER ≡ 1 **cm** ≡ 10^{-2} METERS (≡ 1/100th METER ≡ 0.01 METERS)

1 **deci**METER ≡ 1 **dm** ≡ 10^{-1} METERS (≡ 1/10th METER ≡ 0.1 METERS)

1 **kilo**METER ≡ 1 **km** ≡ 10^{3} METERS (≡ 1,000 METERS)

To help in remembering the meter-stick equalities, visualize a meter stick. Recall what the numbers and marks on a meter stick mean. Use that image to help you to write the equalities above.

To help in remember the kilometer definition, visualize 1,000 meter sticks in a row. That's a distance of one *kilo*meter. 1 kilometer ≡ 1,000 meter sticks.

Once you commit Rule 1 to memory, Rule 2 using the "1-prefix" definition format should be easy to write because it is mathematically equivalent.

For example: To complete 1 centiMETER =

Write 1 METER = 100 centiMETERS

Then to get **1 centi**meter, divide both sides by 100:

1/100 METER = 10^{-2} METER = **1 centi**METER

Rules 1 and 2 are especially important because of Rule

3. You may substitute *any unit* for METER in the equalities above.

Rule 3 means that the prefix relationships that are true for meters are true for *any* units of measure. The three rules above allow us to write a wide range of equalities that we can use to solve science calculations, such as

1 *liter* ≡ 1,000 milli*liters* 1 centi*gram* ≡ 10^{-2} *grams* 1 kilo*calorie* ≡ 10^{3} *calories*

To use *kilo-* , *deci-, centi-* or *milli-* with *any* units, you simply need to be able to write or recall from memory the metric equalities in Rules 1 and 2 above.

Practice A: Write Rules 1 and 2 until you can do so from memory. Learn Rule 3. Then complete these problems without looking back at the rules.

1. From memory, add exponential terms to these blanks.

   a. 1 millimeter = ______meters   b. 1 deciliter = _______ liter

2. From memory, add full metric *prefixes* to these blanks.

   a. 1000 grams = 1 _________gram   b. 10^{-2} liters = 1 ___________liter

Volume

Volume is the amount of three-dimensional space that a material or shape occupies. Volume is termed a **derived quantity**, rather than a fundamental quantity, because it is derived from distance. Any volume unit can be converted to a distance unit cubed.

A cube that is 1 centimeter *wide* by 1 cm *high* by 1 cm *long* has a volume of one **cubic** centimeter (1 **cm**^{3}). In biology and medicine, **cm**^{3} is often abbreviated as "**cc**."

In chemistry, cubic centimeters are usually referred to as **milliliters,** abbreviated as **mL**. One milliliter is defined as exactly one cubic centimeter. Based on this definition, since

- 1,000 milliMETERs ≡ 1 METER, and 1,000 milli*anythings* ≡ 1 *anything*,
- 1,000 milliLITERS is defined as 1 **liter (1 L)**.

The mL is a convenient measure for smaller volumes, while the liter (about 1.1 quarts) is preferred when measuring larger volumes.

One liter is the same as **one cubic deci**m**eter** (1 dm^3). Note how these units are related.

- The volume of a cube that is 10 cm x 10 cm x 10 cm = 1,000 cm^3 = 1,000 mL
- Since 10 cm ≡ 1 dm, the volume of this *same* cube can be calculated as
  1 dm x 1 dm x 1 dm ≡ 1 cubic *deci*meter ≡ 1 dm^3

Based on the above, by definition, all of the following terms are *equal*.

1,000 cm^3 ≡ 1,000 mL ≡ 1 L ≡ 1 dm^3

What do you need to remember about volume? For now, just two more sets of equalities.

4. 1 milliliter (mL) ≡ 1 cm^3 ≡ 1 cc

5. 1 liter ≡ 1,000 mL ≡ 1,000 cm^3 ≡ 1 dm^3

Mass

Mass measures the amount of matter in an object. If you have studied physics, you know that mass and weight are not the same. In chemistry, however, unless stated otherwise, we assume that mass is measured at the constant gravity of the earth's surface. In that case, mass and weight are directly proportional and can be measured with the same instruments.

The metric base-unit for mass is the gram. One **gram** (g) was originally *defined* as the mass of *one cubic centimeter* of *liquid water* at 4° Celsius, the temperature at which water has its highest density. The modern SI definition for one gram is a bit more complicated, but it is still very close to the historic definition. We will often use that historic definition in calculations involving liquid water if high precision is not required.

For a given mass of *liquid* water at 4° C, its volume increases by a small amount with changes in temperature. The volume increases more if the water freezes or boils. However, for *most* calculations for *liquid* water at any temperature, the following rule may be used.

> **6.** 1 $\mathbf{cm^3}$ $H_2O_{(liquid)}$ ≡ 1 **mL** $H_2O_{(l)}$ ≈ 1.00 **gram** $H_2O_{(l)}$ (≈ means *approximately*)

Temperature

Metric temperature scales are defined by the properties of water. The unit of the temperature scale is termed a **degree Celsius** (**ºC)**.

0ºC = the freezing point of water.

100ºC = the boiling point of water at a pressure of one atmosphere.

Room temperature is generally between 20ºC (which is 68ºF) and 25ºC (77ºF).

Time: The base unit for time in the metric system is the **second**.

Unit and Prefix Abbreviations

The following list of abbreviations for metric units should also be committed to memory. Given the unit, you need to be able to write the abbreviation, and given the abbreviation, you need to be able to write the unit.

Unlike other abbreviations, abbreviations for metric units do *not* have periods at the end.

Units: **m** = meter **g** = gram **s** = second

L = liter = $\mathbf{dm^3}$ = cubic decimeter $\mathbf{cm^3}$ = cubic centimeter = **mL** = "cc"

The most frequently used prefixes: k- = kilo- d- = deci- c- = centi- m- = milli-

---

Practice B: Write Rules 3 to 6 until you can do so from memory. Learn the unit and prefix abbreviations as well. Then complete the following problems without looking back at the above.

1. Fill in the prefix abbreviations: 1 m = 10 ____m = 100 ____m = 1000 ____m
2. From memory, add metric prefix *abbreviations* to these blanks.

   a. 10^3 g = 1 ____g b. 10^{-3} s = 1 ___s
3. From memory, add fixed decimal numbers to these blanks.

   a. 1000 cm^3 = _________ mL b. 100 cc $H_2O_{(l)}$ = _______ grams $H_2O_{(l)}$

4. Add fixed decimal numbers: 1 liter ≡ ________ mL ≡ _______ cm^3 ≡ ________ dm^3

SI Units

The modern metric system (*Le **S**ystème **I**nternational d'Unités*) is referred to as the **SI system** and is based on what are termed the **SI units**. SI units are a subset of metric units that chooses **one** preferred metric unit as the standard for measuring each physical quantity.

The **SI standard** unit for distance is the meter, for mass is the kilogram, and for time is the second. Historically, the SI system is derived from what in physics was termed the **mks system** because it measured in units of **m**eters, **k**ilograms, and **s**econds.

In physics, and in many chemistry calculations that are based on relationships derived from physics, using *SI units* is essential to simplify calculations.

However, for dealing with laboratory-scale quantities, chemistry often measures and calculates in units that not SI, but are metric. For example, in chemistry we generally measure mass in grams instead of kilograms. In Modules 4 and 5, you will learn to convert between SI and non-SI units.

Learning the Metric Fundamentals

A strategy that can help in problem-solving is to start each homework assignment, quiz, or test by writing *recently* memorized rules at the top of your paper. By writing the rules at the beginning, you avoid having to remember them under time pressure later in the test.

We will use *equalities* to solve most problems. The 7 metric basics define the equalities that we will use most often.

A Note on Memorization

A goal of these lessons is to minimize what you must memorize. However, it is not possible to eliminate memorization from science courses. When there are facts which you must memorize in order to solve problems, these lessons will tell you. This is one of those times.

Memorize the table of metric basics in the box at the right. You will need to write them automatically, from memory, as part of most assignments in chemistry.

Metric Basics

1. 1 METER ≡ 10 deciMETERS
≡ 100 centiMETERS
≡ 1000 milliMETERS

1,000 METERS ≡ 1 kiloMETER

2. **1 milli**METER ≡ **1 mm** ≡ 10^{-3} METER

1 centiMETER ≡ **1 cm** ≡ 10^{-2} METER

1 deciMETER ≡ **1 dm** ≡ 10^{-1} METER

1 kiloMETER ≡ **1 km** ≡ 10^{3} METER

3. Any word can be substituted for METER above.

4. 1 mL ≡ 1 cm^3 ≡ 1 cc

5. 1 liter ≡ 1000 mL ≡ 1000 cm^3 ≡ 1 dm^3

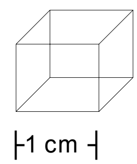
6. 1 cm^3 $H_2O_{(liquid)}$ ≡ 1 mL $H_2O_{(l)}$
≈ 1.00 gram $H_2O_{(l)}$

7. meter ≡ m ; gram ≡ g ; second ≡ s

Memorization Tips

When you memorize, it helps to use as many *senses* as you can.

- *Say* the rules out loud, over and over, as you would learn lines for a play.
- *Write* the equations several times, in the same way and order each time.
- *Organize* the rules into patterns, rhymes, or mnemonics.
- *Number* the rules so you know which rule you forgot, and when to stop.
- *Picture* real objects.
    - o Sketch a meter stick, then write the first two metric rules and compare to your sketch.
    - o Write METER in ALL CAPS for the first two rules as a reminder that you that you can substitute ANYTHING for METER.
    - o For volume, mentally picture a 1 cm x 1 cm x 1 cm = ***1 cm***^{3} cube. Call it *one mL*. Fill it with water to make a *mass* of 1.00 *grams***.**

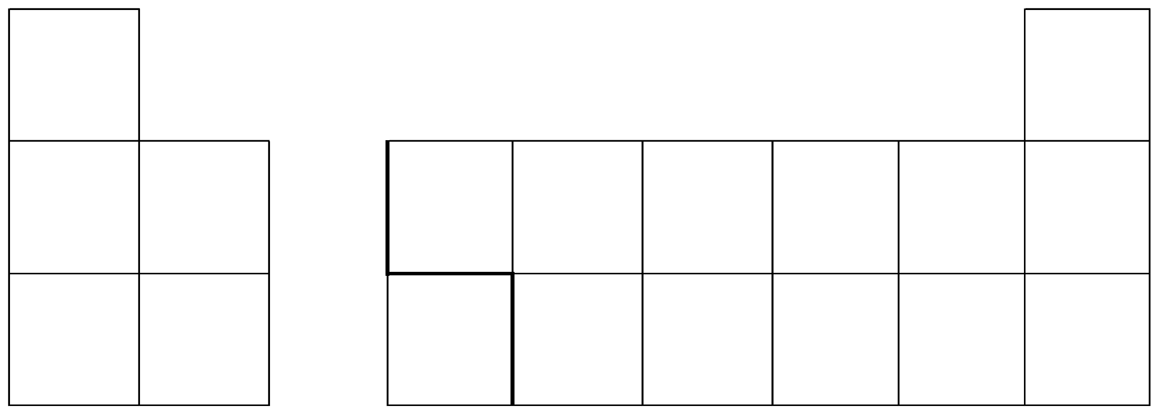


After repetition, you will recall new rules *automatically*. That's the goal.

Practice C: Study the 7 rules in the *metric basics* table above, then write the table on paper from memory. Repeat until you can write all parts of the table from memory, 100%. Then cement your knowledge by doing these problems. Check your answers below.

1. In your mind, picture a kilometer and a millimeter. Which is larger?
2. Which is larger, a kilojoule or a millijoule?
3. Name four units that can be used to measure volume in the metric system.
4. How many centimeters are on a meter stick?
5. How large is a kiloliter?
6. What is the mass of 15 milliliters of liquid water?
7. One liter of liquid water has what mass?
8. What is the volume of one gram of ice?

9. Fill in the portion of the Periodic Table below for the first 12 atoms.



ANSWERS

Pretest: 1. 0.15 kg 2. 1,000 cm^3, 1 dm^3 3. 2,500 millipascals 4. 0.035 kg

Practice A

1a. 1 millimeter = $\mathbf{10^{-3}}$ meters

1b. 1 deciliter = $\mathbf{10^{-1}}$ liter

2a. 1000 grams = 1**kilo**gram

2b. 10^{-2} liters = 1 **centi**liter

Practice B

1. 1 m = 10 **d**m = 100 **c**m = 1000 **m**m

2a. 10^{3} g = 1 **k**g

2b. 10^{-3} s = 1 **m**s

3a. 1000 cm^3 = **1000** mL

3b. 100 cc $H_2O_{(l)}$ = **100** grams $H_2O_{(l)}$

4. 1 liter ≡ **1000** mL ≡ **1000** cm^3 ≡ **1** dm^3

Practice C

1. A kilometer 2. A kilojoule
3. Possible answers include cubic centimeters, milliliters, liters, cubic decimeters, cubic meters, and any metric distance unit cubed.
4. 100 5. 1,000 liters 6. 15 grams 7. 1,000 grams *or* one kilogram
8. These lessons have not supplied the answer. Water expands when it freezes. So far, we only know the answer for liquid water.
9. See Periodic Table.

* * * * *

Lesson 2B: Metric Prefixes

Pretest: If you have previously mastered use of the prefixes in the table below, try the Practice B problems at the end of this lesson. If you get those right, you may skip this lesson.

* * * * *

Additional Prefixes

For measurements of very large or very small quantities, prefixes larger than *kilo*- and smaller than *milli*- may be used. The 13 prefixes encountered most frequently are listed in the table at the right. Note that

- Outside the range between −3 and 3, metric prefixes are abbreviations of powers of 10 that are divisible by **3**.
- When the full prefix name is written, the first letter is not normally capitalized.
- For prefixes above ***k-*** (*kilo*-), the *abbreviation* for a prefix *must* be *capitalized*.
- For the prefixes ***k-*** and below, all letters of the abbreviation *must* be lower case.

Prefix	Abbreviation	Means
tera-	T-	x 10^{12}
giga-	G-	x 10^{9}
mega-	M-	x 10^{6}
kilo-	k-	x 10^{3}
hecto-	h-	x 10^{2}
deka-	da-	x 10^{1}
deci-	d-	x 10^{-1}
centi-	c-	x 10^{-2}
milli-	m-	x 10^{-3}
micro-	µ- (mu) *or* u-	x 10^{-6}
nano-	n-	x 10^{-9}
pico-	p-	x 10^{-12}
femto-	f-	x 10^{-15}

Using Prefixes

A metric prefix is interchangeable with the exponential term it represents. For example, during measurements and/or calculations,

- An *exponential* term can be *substituted* for its equivalent metric prefix.

  Examples: 7.0 *milli*liters = 7.0 **x 10^{-3}** liters

  5.6 kg = 5.6 **x 10^{3}** g

  43 nanometers = 43 nm = 43 **x 10^{-9}** m

- A metric *prefix* can be substituted for its equivalent exponential term.

  Examples: 3.5 x 10^{-12} meters = 3.5 **pico**meters = 3.5 pm

  7.2 x 10^{6} watts = 7.2 **mega**watts

In calculations, we will often need to convert between a prefix and its equivalent exponential term. One way to do this is to apply the prefix definitions.

Q1. From memory, fill in these blanks with prefixes.

a. 10^3 grams = 1 _________gram   b. 2 x 10^{-3} meters = 2 ________ meters

Q2. From memory, fill in these blanks with prefix *abbreviations*.

a. 2.6 x 10^{-1} L = 2.6 ____L   b. 6 x 10^{-2} g = 6 ____g

Q3. Fill in these blanks with exponential terms (use the table above *if* needed).

a. 1 gigajoule = 1 x _______ joules   b. 9 μm = 9 x ________ m

* * * * *

Answers

1a. 10^3 grams = 1 **kilo**gram   1b. 2 x 10^{-3} meters = 2 **milli**meters

2a. 2.6 x 10^{-1} L = 2.6 **d**L.   2b. 6 x 10^{-2} g = 6 **c**g

3a. 1 gigajoule = 1 x $\mathbf{10^9}$ joules   3b. 9 μm = 9 **x** $\mathbf{10^{-6}}$ m

From the prefix definitions, even if you are not yet familiar with the quantity that a unit is measuring, you can convert between its *prefix*-unit value and its value using exponentials.

Science Versus Computer-Science Prefixes

Computer science, which calculates based on powers of 2, uses slightly different definitions for prefixes, such as *kilo*- = 2^{10} = 1,024 instead of 1,000.

However, in chemistry and all other sciences, for all base units, the prefix to power-of-10 relationships in the metric-prefix table are *exact* definitions.

Learning the Additional Prefixes

To solve calculations, you will need to recall the rows in the table of 13 metric prefixes quickly and automatically. To begin, practice writing the table from memory. To help, look for patterns and use memory devices. Note

- **t**era = **T** = 10^{Twelve} and **n**ano (which connotes *small*) = $\mathbf{10^{-\text{nine}}}$.

Focusing on those two can help to "anchor" the prefixes near them in the table.

Then make a self-quiz: on a sheet of paper, draw a table 3 columns across and 14 rows down. In the top row, write

Prefix	**Abbreviation**	**Means**

Then fill in the table. Repeat writing the table until you can do so from memory, without looking back. Once you can do so, try to do the problems below without looking back at your table.

Practice A: Use a sticky note to mark the answer page at the end of this lesson.

1. From memory, add exponential terms to these blanks.

   a. 7 microseconds = 7 x ______ seconds b. 9 fg = 9 x ________ g

   c. 8 cm = 8 x ________ m d. 1 ng = 1 x ________ **g**

2. From memory, add full metric *prefixes* to these blanks.

   a. 6×10^{-2} amps = 6 ____________ amps b. 45×10^{9} watts = 45 ________watts

3. From memory, add metric prefix *abbreviations* to these blanks.

   a. 10^{12} g = 1 ____g b. 10^{-12} s = 1 ___s

   c. 6×10^{-9} m = 6 ____ m d. 5×10^{-1} L = 5 ____ L

   e. 4×10^{1} L = ___________ L f. 16×10^{6} Hz =16 _________Hz

4. When writing prefix abbreviations *by hand*, write so that you can distinguish between (add a prefix abbreviation) 5×10^{-3} g = 5 ____g and 5×10^{6} g = 5 ____g

5. For which prefix abbreviations is the first letter always capitalized?

6. Write 0.30 gigameters/second without a prefix, in scientific notation.

Converting Between Prefix Formats

To solve calculations in chemistry, we will often use conversion factors that are constructed from metric prefix definitions. For those definitions, we have learned two types of equalities.

- Our "meter stick" equalities are based on what *one unit* is equal to:

1 METER ≡ 10 **deci**METERS ≡ 100 **centi**METERS ≡ 1,000 **milli**METERS

- Our prefix definitions are based on what *one prefix* is equal to, such as *nano* = 10^{-9} .

It is essential to be able to correctly write *both* forms of the metric definitions, because work in science often uses both.

For example, to convert between milliliters and liters, we can use *either*

- 1 mL = $\mathbf{10^{-3}}$ L , based on what *1 milli-* means, *or*
- 1,000 mL = 1 L ; which is an easy-to-visualize definition of one liter.

Those two equalities are equivalent. The second is simply the first with the numbers on both sides multiplied by 1,000.

However, note that **1** mL = 10^{-3} L , but **1** L = 10^{3} mL . The numbers in the equalities change depending on whether the 1 is in front of the prefix or the unit. Which format should we use? How do we avoid errors?

In these lessons, we will generally use the *one prefix* equalities to solve problems. After learning the fundamental definitions for the 10 prefixes in the table, such as 1 milli- = **10^{-3}**, using the definitions makes conversions easy to check.

Once those prefix, abbreviation, and meanings are in memory, we will then need to "watch where the 1 is."

If you need to write or check prefix equalities in the "one **unit** =" format, you can derive them from the *one prefix* definitions, by writing the table if needed.

<u>For example,</u> 1 gram = ______ **micrograms**?

- Since 1 **micro**-anything = **10^{-6}** anythings, then
- 1 **microgram** = **10^{-6}** grams
- To get a 1 in front of gram, we multiply both sides by **10^{6}**, so
- **1 gram = 10^{6} micrograms** (**= 10^{6} μg =** 1,000,000 micrograms **)**

The steps above can be summarized as the *reciprocal* rule for prefixes:

If **1 prefix- = 10^{a}** , 1 ***unit*** = **10^{-a}** *prefix*-units

Another way to state the reciprocal rule for prefixes:

To change a prefix definition between the "1 prefix- = " format and the "1 unit = " format, change the sign of the exponent.

If you need to check your logic, write the most familiar example:

Since 1 **milli**liter = **10^{-3}** liter , then 1 liter = **10^{3}** milliliters = 1,000 mL

Try these examples.

Q1. 1 nanogram = 1 x ______ grams , so 1 gram = 1 x ______ nanograms

Q2. 1 dL = 1 x ______ liters , so 1 L = 1 x ______ dL

* * * * *

<u>Answers</u>

A1. 1 nanogram = 1 x **10^{-9}** grams , so 1 gram = 1 x **10^{9}** nanograms

A2. 1 dL = 1 x **10^{-1}** liters , so 1 L = 1 x **10^{1}** dL = 10 dL

<u>To summarize:</u>

- When using metric prefix definitions, be careful to note whether the **1** is in front of the prefix or the unit.

- To avoid confusing the signs of the exponential terms in prefix definitions, memorize the table of 13 *one prefix* definitions. Then, if you need an equality with a "*1 unit* = $\mathbf{10^x}$ prefix-unit" format, reverse the sign of the prefix definition.

Practice B: Write the table of the 13 metric prefixes until you can do so from memory, then try to do these without consulting the table.

1. Fill in the blanks with exponential terms.

   a. 1 terasecond = 1 x ______ seconds , so 1 second = 1 x ______ teraseconds

   b. 1 μg = 1 x ______ grams , so 1 g = 1 x ______ μg

2. Apply the reciprocal rule to add exponential terms to these *one unit* equalities.

   a. 1 gram = _______ centigrams b. 1 meter = ___________ picometers

   c. 1 s = ___________ ms d. 1 s = ___________ Ms

3. Add exponential terms to these blanks. Watch where the 1 is!

   a. 1 micromole = _____ moles b. 1 g = 1 x _______ Gg

   c. 1 hectogram = 1 x _______ grams d. _______ kg = 1 g

   e. _______ ns = 1 s f. 1 fL = ________ L

ANSWERS

Practice A

1. a. 7 microseconds = 7 x $\mathbf{10^{-6}}$ seconds b. 9 fg = 9 x $\mathbf{10^{-15}}$ g

   c. 8 cm = 8 x $\mathbf{10^{-2}}$ m d. 1 ng = 1 x $\mathbf{10^{-9}}$ **g**

2. a. 6 x 10^{-2} amps = 6 **centi**amps b. 45 x 10^9 watts = 45 **giga**watts

3. a. 10^{12} g = 1 **T**g b. 10^{-12} s = 1 **p**s

   c. 6 x 10^{-9} m = 6 **n**m d. 5 x 10^{-1} L = 5 **d**L

   e. 4 x 10^1 L = 4 **da**L f. 16 x 10^6 Hz =16 **M**Hz

4. 5 **m**g and 5 **M**g 5. **M-**, **G-**, and **T-.** 6. **3.0 x** $\mathbf{10^8}$ meters/second

Practice B

1. a. 1 terasecond = 1 x $\mathbf{10^{12}}$ seconds , so 1 second = 1 x $\mathbf{10^{-12}}$ teraseconds

   b. 1 μg = 1 x $\mathbf{10^{-6}}$ grams , so 1 g = 1 x $\mathbf{10^6}$ μg

2. a. 1 gram = $\mathbf{10^2}$ centigrams (For " 1 *unit* = ", take reciprocal (reverse sign) of prefix meaning)

   b. 1 meter = $\mathbf{10^{12}}$ picometers c. 1 s = $\mathbf{10^3}$ ms d. 1 s = 1 x $\mathbf{10^{-6}}$ Ms

3. a. 1 micromole = $\mathbf{10^{-6}}$ moles b. 1 g = 1 x $\mathbf{10^{-9}}$ Gg

c. 1 hectogram = 1 x 10^2 grams
d. 10^{-3} kg = 1 g
e. 10^9 ns = 1s
f. 1 fL = 10^{-15} L

* * * * *

Lesson 2C: Cognitive Science – and Flashcards

In this lesson, you will learn a *system* that will help you to automatically recall the vocabulary needed to read science with comprehension and the facts needed to solve calculations.

Cognitive science studies how the mind works and how it learns. The model that science uses to describe learning includes the following fundamentals.

- The purpose of learning is to solve problems. You solve problems using information from your immediate environment and your memory.

The human brain contains different types of memory, including

- **Working** memory: the part of your brain where you solve problems.
- **Short-term** memory: information that you remember for only a few days.
- **Long-term** memory: information that you can recall for many years.

Working memory is limited, but human *long*-term memory has enormous capacity. The goal of learning is to move new information from short into long-term memory so that it can be recalled by working memory for years after initial study. If information is not moved into long-term memory, useful long-term learning has not taken place.

Children learn speech naturally, but most other learning requires repeated *thought* about the meaning of new information, plus *practice* at recalling new facts and using new skills that is *timed* in ways that encourage the brain to move new learning from short to long-term memory.

The following principles of cognitive science will be helpful to keep in mind during your study of chemistry and other disciplines.

1. **Learning is cumulative.** Experts in a field learn new information quickly because they already have in long-term memory a storehouse of knowledge about the context surrounding new information. That storehouse must be developed over time, with practice.

2. **Learning is incremental** (done in small pieces)**.** Especially for an unfamiliar subject, there is a limit to how much new information you can move into long-term memory in a short amount of time. Knowledge is extended and refined gradually. In learning, *steady* wins the race.

3. **Your brain can do parallel processing.** Though adding information to long term memory is a gradual process, studies indicate that your brain can work on separately remembering what something looks like, where you saw it, what it sounds like, how you say it, how you write it, and what it means, all at the same time. The cues associated with each separate type of memory can help to trigger the recall of information needed to solve a problem, so it helps to use multiple strategies. When

learning new information: listen to it, see it, say it, write it, and try to connect it to other information that helps you to remember what it means.

4. **The working memory in your brain is limited.** Working memory is where you think. Try multiplying 556 by 23 in your head. Now try it with a pencil, a paper, and your head. Because of limitations in working memory, manipulating multiple pieces of new information "in your head" is difficult. Learning stepwise procedures (standard algorithms) that write the results of middle steps is one way to reduce "cognitive load" during problem solving.

5. **"Automaticity in the fundamentals"** is another learning strategy that can help to overcome limitations in working memory. When you can recall facts quickly due to repeated practice, more working memory is available for higher level thought.

   You can do work that is *automatic* while you think (most of us can think while walking), but it is difficult to *think* about more than one problem at once.

6. **Concepts are crucial.** Your brain works to construct a "conceptual framework" to categorize knowledge being learned so that you can recall facts and procedures when you need them. The brain tends to store information in long-term memory only if it is in agreement with your framework of concepts. In addition, if you have a more complete and accurate understanding of "the big picture," your brain is better able to judge which information should be selected to solve a problem.

   Concepts do not replace the need to move key facts and procedures into your long-term memory, but concepts speed initial learning, recall, and appropriate application of your knowledge in long-term memory.

7. **"You can always look it up" is a poor strategy for problem-solving.** Your working memory is quite limited in how much information it can manipulate that is not in your long-term memory. The more information you must stop to look up, the less likely you will be able to follow your train of thought to the end of a complex problem.

How can you promote the retention of needed fundamentals? It takes practice, but some forms of practice are more effective than others. Attention to the following factors can improve your retention of information in long-term memory.

1. **Overlearning.** Practice once until you are perfect and you will only recall new information for a few days. To be able to recall new facts and skills for more than a few days, *repeated* practice to perfection is necessary.

2. **The spacing effect.** To *retain* what you learn, 20 minutes of study spaced over 3 days is more effective than one hour of study for one day.

   Studies of "massed versus distributed practice" show that if the initial learning of facts and vocabulary is practiced over 3-4 days, then re-visited weekly for 2-3 weeks, then monthly for 3-4 months, it can often be recalled for decades thereafter.

3. **Effort.** Experts in a field usually attribute their success to "hard work over an extended period of time" rather than "talent."

4. **Core skills.** The facts and processes you should practice most often are those needed most often in the discipline.

5. **Get a good night's sleep.** There is considerable evidence that while you sleep, your brain reviews the experience of your day to decide what to store in long-term memory. Sufficient sleep promotes retention of what you learn.

[For additional science that relates to learning, see Willingham, Daniel [2007] *Cognition: The Thinking Animal.* Prentice Hall, and Bruer, John T. [1994] *Schools for Thought*. MIT Press.]]

Practice A

1. What is "overlearning?"
2. What is the "spacing effect?"
3. What are two learning strategies that can help to overcome inherent limitations on the manipulation of new information in your working memory?

Flashcards

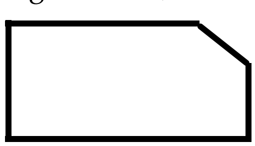
What is more important in learning: Knowing the facts or the concepts? Cognitive studies have found that the answer is: both. However, to "think as an expert," you need a storehouse of factual information in memory that you can apply to new and unique problems.

In these lessons, we will use the following flashcard system to master fundamentals that need to be recalled automatically in order to efficiently solve problems. Using this system, you will make two types of flashcards:

- "One-way cards" for questions that make sense in *one* direction; and
- "Two-way" cards for facts that need to be recalled in both directions.

If you have access to about 30 3 x 5 index cards, you can get started now. Plan to buy tomorrow about 100-200 3x5 index cards, lined or unlined. (A variety of colors is helpful but not essential.) Complete these steps.

1. On 12-15 of your 30 initial cards (of the same color if possible), cut a triangle off the top-right corner, making cards like this:



These cards will be used for questions that go in *one* direction.

Keeping the notch at the *top right* will identify the *front* side.

2. Using the following table, cover the *answers* in the right column with a folded sheet or index card. For each question in the left column, verbally answer, then slide the cover sheet down to check your answer. Put a check beside questions that you answer accurately and without hesitation. When done, write the questions and answers with*out* checks onto the notched cards.

Front-side of cards (with notch at top right):	**Back Side -- Answers**
To convert to scientific notation, move the decimal to…	After the first number not a zero
If you make the significand larger	Make the exponent smaller
42^0	Any number to the zero power = 1

To *add* or *subtract* in exponential notation	Make all exponents the same
Simplify $1/10^{-x}$	10^{x}
To divide exponentials	Subtract the exponents
To bring an exponent from the bottom of a fraction to the top	Change its sign
1 cc ≡ 1 ___ ≡ 1 ___	1 cc ≡ 1 cm^3 ≡ 1 mL
0.0018 in scientific notation =	1.8×10^{-3}
1 L ≡ ___ mL ≡ ___ dm^3	1 L ≡ 1,000 mL ≡ 1 dm^3
To multiply exponentials	Add the exponents
Simplify $1/10^{x}$	10^{-x}
74 in scientific notation =	7.4×10^{1}
The historic definition of 1 gram	The mass of 1 cm^3 of liquid water at 4ºC.
8 x 7	56
42/6	7

Any multiplication or division up to 12's that you cannot answer *instantly*? Add to your list of one-sided cards. If you need a calculator to do number math, parts of chemistry such as "balancing an equation" will be frustrating. With flashcard practice, you will quickly be able to remember what you need to know.

3. To make "two-way" cards, use the index cards as they are, with*out* a notch cut.

For the following cards, first cover the *right* column, then put a check on the left if you can answer the left column question *quickly* and correctly. Then cover the *left* column and check the right side if you can answer the right-side *automatically*.

When done, if a row does not have *two* checks, make the flashcard.

Two-way cards (with*out* a notch):

10^3 g or 1,000 g = 1 __g	1 kg = ____ g
Boiling temperature of water	100 degrees Celsius -- if 1 atm. pressure
1 nanometer = 1 x ___ meters	1 ____meter = 1×10^{-9} meters
Freezing temperature of water	0 degrees Celsius
4.7×10^{-3} = ______________(nbr)	0.0047 = $4.7 \times 10^{?}$

1 GHz =$10^{?}$ Hz	10^9 Hz = 1 __Hz
1 pL = $10^{?}$ L	10^{-12} L = 1 __L
3/4 = 0.?	0.75 = ? / ?
1/8 = 0.?	0.125 = 1 / ?

2/3 = 0.?	0.666… = ? / ?
1/80 = 0.?	0.**0**125 = 1 / ?
1 dm^3 = 1 ___	1 L = 1 __
1/4 = 0.?	0.25 = 1 / ?

More two-way cards (with*out* a notch) for the metric-prefix definitions.

kilo = x $10^{?}$	x 10^{3} = ? Prefix
nano = x $10^{?}$	x 10^{-9} = ? pref.
giga = x $10^{?}$	x 10^{9} = ? Prefix
milli = x $10^{?}$	x 10^{-3} = ? pref.
deci = x $10^{?}$	x 10^{-1} = ? pref.
tera = x $10^{?}$	x 10^{12} = ? pref.
pico = x $10^{?}$	x 10^{-12} = ? pref
hecto = x $10^{?}$	x 10^{2} = ? Prefix
deka = x $10^{?}$	x 10^{1} = ? Prefix
femto = x $10^{?}$	x 10^{-15} = ? pref
mega = x $10^{?}$	x 10^{6} = ? Prefix
micro = x $10^{?}$	x 10^{-6} = ? pref.
centi = x $10^{?}$	x 10^{-2} = ? pref.

d = x $10^{?}$	x 10^{-1} = ? abbr.
m = x $10^{?}$	x 10^{-3} = ? abbr.
T = x $10^{?}$	x 10^{12} = ? abbr.
k = x $10^{?}$	x 10^{3} = ? abbr.
f = x $10^{?}$	x 10^{-15} = ? abb
μ = x $10^{?}$	x 10^{-6} = ? abbr.
G = x $10^{?}$	x 10^{9} = ? abbr.
da = x $10^{?}$	x 10^{1} = ? abbr.
p = x $10^{?}$	x 10^{-12} = ? abb
c = x $10^{?}$	x 10^{-2} = ? abbr.
h = x $10^{?}$	x 10^{2} = ? abbr.
M = x $10^{?}$	x 10^{6} = ? abbr.
n = x $10^{?}$	x 10^{-9} = ? abbr.

micro =? abbr.	μ = ? pref.
mega =? abbr.	M = ? pref.
deka =? abbr.	da = ? pref.
pico =? abbr.	p = ? prefix
deci =? abbr.	d = ? prefix
hecto =? abbr.	h = ? prefix
tera =? abbr.	T = ? prefix
milli =? abbr.	m = ? pref.
femto =? abbr.	f = ? prefix
giga =? abbr.	G = ? pref.
nano =? abbr.	n = ? prefix
centi =? abbr.	c = ? prefix
kilo =? abbrev.	k = ? prefix

Which cards you need will depend on your prior knowledge, but when in doubt, make the card. On fundamentals, you need quick, confident, accurate recall -- every time.

4. **Practice** with one *type* of card at a time.
   - **For front-sided cards**, if you get a card right quickly, place it in the *got it* stack. If you miss a card, say it. Close your eyes. Say it again. And again. If needed, write it several times. Return that card to the bottom of the *do* deck. Practice until every card is in the *got-it* deck.
   - **For two-sided cards,** do the same steps as above in one direction, then the other.
5. Master the cards at least once, *then* apply them to the **Practice** on the topic of the new cards. Treat **Practice** as a practice *test*.
6. **For 3 days in a row,** repeat those steps. Repeat again before working assigned problems, before your next quiz, and before your next test that includes this material.
7. Make cards for new topics early: before the lectures on a topic if possible. Mastering fundamentals first will help in understanding lecture.
8. Rubber band and carry new cards. Practice during "down times."
9. After a few modules or topics, change card colors.

This system requires an initial investment of time, but in the long run it will save time and improve achievement.

The above flashcards are examples. Add cards of your design and choosing as needed.

Flashcards, Charts, or Lists?

What is the best strategy for learning new information? Use *multiple* strategies: numbered lists, mnemonics, phrases that rhyme, flashcards, reciting, and writing what must be remembered. Practiced repeatedly, spaced over time.

For complex information, automatic recall may be less important than being able to methodically write out a *chart* for information that falls into *patterns*.

For the metric system, learning flashcards *and* the prefix chart *and* picturing the meter-stick relationships all help to fix these fundamentals in memory.

Practice B: Run your set of flashcards until all cards are in the "got-it" pile. *Then* try these problems. Make additional cards if needed. Run the cards again in a day or two.

1. Fill in the blanks.

Format: 1 prefix-	1 base unit
1 µMETER = ______ METERS	1 METER = _______ µMETERS
1 gigawatt = ______ watts	1 watt = ________ gigawatts
1 nanoliter = ______ liter	_______ nanoliters = 1 liter

2. Add exponential terms to these blanks. Watch where the 1 is!

 a. 1 picocurie = ___________ curies   b. 1 megawatt = ___________watts

 c. 1 dag = ____________ g   d. 1 mole = ___________ millimoles

 e. 1 m = ___________ nm   f. 1 kPa = ____________ Pa

3. Do these without a calculator.

 a. $10^{-6}/10^{-8}$ =   b. 1/5 = ___.___ ___   c. 1/50 = ___.___ ___ ___

4. For the following atoms, write the symbol.

 a. Helium = _______   b. Hydrogen = ________   c. Sodium = __________

5. For the following symbols, write the atom name.

 a. N = _______________   b. Ne = ______________   c. B = _____________

ANSWERS

Practice A

1. Repeated practice to perfection. 2. Study over several days gives better retention than "cramming."
3. Learning stepwise procedures (algorithms) and learning fundamentals until they can be recalled automatically.

Practice B

1.

1 μMETER = **10^{-6}** METERS	1 METER = **10^{6}** μMETERS
1 gigawatt = **10^{9}** watts	1 watt = **10^{-9}** gigawatts
1 nanoliter = **10^{-9}** liters	**10^{9}** nanoliters = 1 liter

2. a. 1 picocurie = **10^{-12}** curies b. 1 megawatt = **10^{6}** watts c. 1 dag = **10^{1}** g

   d. 1 mole = **10^{3}** millimoles e. 1 m = **10^{9}** nm f. 1 kPa = **10^{3}** Pa

3. a. $10^{-6}/10^{-8} = 10^{-6+8} = \mathbf{10^{2}}$ b. 1/5 = **0.20** c. 1/50 = **0.020**

4a. Helium = **He** 4b. Hydrogen = **H** 4c. Sodium = **Na**

5a. N = **Nitrogen** 5b. Ne = **Neon** 5c. B = **Boron**

* * * * *

Lesson 2D: Calculations With Units

Pretest: If you can do the following two problems correctly, you may skip this lesson. Answers are at the end of the lesson.

1. Find the volume of a sphere that is 4.0 cm in diameter. ($V_{sphere} = 4/3\pi r^3$).
2. Multiply: $2.0 \frac{g \cdot m}{s^2} \cdot \frac{3.0 \text{ m}}{4.0 \times 10^{-2}} \cdot 6.0 \times 10^2 \text{ s} =$

* * * * *

(Try doing this lesson with*out* a calculator except as noted.)

Adding and Subtracting With Units

Many calculations in mathematics consist of numbers without units. In science, however, calculations are nearly always based on measurements of physical quantities. A measurement consists of a numeric value *and* its unit.

When doing calculations in science, it is essential to write the *unit* after the numbers in measurements and calculations. Why?

- Units give physical mean to a quantity.
- Units are the best indicators of what steps are needed to solve problems, and
- Units provide a check that you have done a calculation correctly.

When solving calculations, the math must take into account *both* the numbers and their units. Use the following three rules.

> **Rule 1.** When *adding* or *subtracting*, the *units must* be the *same* in the quantities being added and subtracted, and those same units must be added to the answer.

Rule 1 is logical. Apply it to these two examples.

A. 5 apples + 2 apples = _________ B. 5 apples + 2 oranges = __________

* * * * *

A is easy. B cannot be added. It makes sense that you *can* add two numbers that refer to apples, but you *can't* add apples and oranges. By Rule 1, you can add numbers that have the same units, but you can*not* add numbers directly that do *not* have the same units.

Apply Rule 1 to this problem:

$$\begin{array}{r} 14.0 \text{ grams} \\ -\ \underline{\ \ 7.5 \text{ grams}} \end{array}$$

* * * * *

$$\begin{array}{r} 14.0 \text{ grams} \\ -\ \underline{\ \ 7.5 \text{ grams}} \\ \mathbf{6.5}\ \boldsymbol{grams} \end{array}$$

If the units are all the same, you can add or subtract numbers, but you must add the common unit to the answer.

Multiplying and Dividing With Units

The rule for *multiplying* and *dividing* with units is different, but logical.

> **Rule 2.** When multiplying and dividing *units*, the units multiply and divide.

Complete this example of unit math: cm x cm = ________ .

* * * * *

cm x cm = $\mathbf{cm^2}$ Units obey the laws of algebra. Try: $\frac{cm^5}{cm^2}$ = _____________

* * * * *

$\frac{cm^5}{cm^2}$ = can be solved as $\frac{cm \cdot cm \cdot cm \cdot \cancel{cm} \cdot \cancel{cm}}{\cancel{cm} \cdot \cancel{cm}} = \mathbf{cm^3}$

or by using the rules for exponential terms:

$\frac{cm^5}{cm^2} = cm^{5-2} = \mathbf{cm^3}$ Both methods arrive at the same answer (as they must).

> **Rule 3.** When multiplying and dividing, *group* numbers, exponentials, and units separately. Solve the three parts separately, then recombine the terms.

Apply Rule 3 to this problem: If a postage stamp has the dimensions 2.0 cm x 4.0 cm, the surface area of one side of the stamp = ____________

* * * * *

Area of a rectangle = $l \times w$ =

= 2.0 cm x 4.0 cm = (2.0 x 4.0) x (cm x cm) = **8.0 cm²** = 8.0 *square* centimeters

By Rule 2, the units must obey the rules of multiplication and division. By Rule 3, the unit math is done *separately* from the number math.

Units follow the familiar laws of multiplication, division, and powers, including "like units cancel."

Apply Rule 3 to these: a. $\frac{8.0\ L^6}{2.0\ L^2}$ = __________ b. $\frac{9.0\ m^6}{3.0\ m^6}$ = _________

* * * * *

a. $\frac{8.0\ L^6}{2.0\ L^2} = \frac{8.0}{2.0} \bullet \frac{L^6}{L^2} = 4.0\ \mathbf{L^4}$ b. $\frac{9.0\ \cancel{m^6}}{3.0\ \cancel{m^6}} = \mathbf{3.0}$ (with no unit.)

In science, the *unit math* must be done as part of calculations. A *calculated* unit *must* be included as part of calculated answers (except in rare cases, such as *part b* above, when all of the units cancel).

On the following problem, apply separately the math rules for numbers, exponential terms, and units.

$$\frac{12 \times 10^{-3}\ m^4}{3.0 \times 10^2\ m^2} = \_\_\_\_\_\_\_\_\_\_\_\_$$

* * * * *

$$\frac{12 \times 10^{-3}\ m^4}{3.0 \times 10^2\ m^2} = \frac{12}{3.0} \bullet \frac{10^{-3}}{10^2} \bullet \frac{m^4}{m^2} = \mathbf{4.0 \times 10^{-5}\ m^2}$$

When calculating, you often need to use a calculator to do the number math, but both the *exponential* and *unit* math nearly always should be done with*out* a calculator.

In the problems above, the units were all the same. However, units that are *different* can also be multiplied and divided by the usual laws of algebra. Complete this calculation:

$$4.0\ \frac{g \bullet m}{s^2} \bullet 3.0\ m \bullet \frac{6.0\ \ s}{9.0 \times 10^{-4}\ m^2} =$$

* * * * *

When multiplying and dividing, do the *number, exponential*, and *unit* math separately.

$$4.0\ \frac{g \bullet m}{s^2} \bullet 3.0\ m \bullet \frac{6.0\ \ s}{9.0 \times 10^{-4}\ m^2} = \frac{72}{9.0} \bullet \frac{1}{10^{-4}} \bullet \frac{g \bullet \cancel{m}}{s \bullet \cancel{s}} \bullet \cancel{m} \bullet \frac{\cancel{s}}{\cancel{m^2}} = \mathbf{8.0 \times 10^4\ \frac{g}{s}}$$

This answer unit can also be written as $\mathbf{g \bullet s^{-1}}$, but you will find it helpful to use the **x/y** unit format until we work with mathematical equations later in the course.

Practice: Do *not* use a calculator except as noted. If you need just a few reminders, do Problems 11 and 14. If you need more practice, do more. After completing each problem, check your answer below. If you miss a problem, review the rules to figure out why before continuing.

1. 16 cm − 2 cm =
2. 12 cm • 2 cm =
3. $(m^4)(m)$ =
4. m^4 / m =
5. $\frac{10^5}{10^{-2}}$ =
6. $\frac{s^{-5}}{s^2}$ =
7. 3.0 meters • 9.0 meters =
8. 3.0 g / 9.0 g =
9. $\frac{24\ L^5}{3.0\ L^{-4}}$ =
10. $\frac{18 \times 10^{-3}\ g \bullet m^5}{3.0 \times 10^1\ m^2}$ =
11. $12 \times 10^{-2}\ \frac{L \bullet g}{s} \bullet 2.0\ m \bullet \frac{4.0\ s^3}{6.0 \times 10^{-5}\ L^2}$ =
12. A rectangular box has dimensions of 2.0 cm x 4.0 cm x 6.0 cm. Without a calculator, calculate its volume.
13. Do pretest problem 1 at the beginning of this lesson (use a calculator).
14. Do pretest problem 2 at the beginning of this lesson (without a calculator).

ANSWERS

Both the number *and* the ***unit*** must be written and correct.

Pretest: See answers to Problems 13 and 14 below.

1. **14 cm** 2. **24 cm²** 3. $m^{(4+1)}$ = $\mathbf{m^5}$ 4. $m^{(4-1)}$ = $\mathbf{m^3}$ 5. $\mathbf{10^7}$

6. $\mathbf{s^{-7}}$ 7. **27 meters²** 8. **0.33 (*no* unit)** 9. $\mathbf{8.0\ L^9}$ 10. $\mathbf{6.0 \times 10^{-4}\ g \bullet m^3}$

11. $\mathbf{16 \times 10^3\ \frac{g \bullet m \bullet s^2}{L}}$ 12. $V_{rectangular\ solid}$ = length *times* width *times* height = **48** cm^3

13. Diameter = 4.0 cm, *radius* = 2.0 cm.

$V_{sphere} = 4/3\ \pi\ r^3 = 4/3\ \pi\ (2.0\ cm)^3 = 4/3\ \pi\ (8.0\ cm^3) = \mathbf{(32/3)\ \pi\ cm^3} = \mathbf{34}\ cm^3$

14. $2.0\ \frac{g \bullet m}{s^2} \bullet \frac{3.0\ m}{4.0 \times 10^{-2}} \bullet 6.0 \times 10^2\ s = \frac{(2.0)(3.0)(6.0)}{4.0} \bullet 10^4 \bullet \frac{g \bullet m \bullet m \bullet s}{s^2} = \mathbf{9.0 \times 10^4\ \frac{g \bullet m^2}{s}}$

* * * * *

SUMMARY – The Metric System

Prefix	Abbreviation	Means
tera-	T	x 10^{12}
giga-	G	x 10^{9}
mega-	M	x 10^{6}
kilo-	k	x 10^{3}
hecto-	h	x 10^{2}
deka-	da	x 10^{1}
deci-	d	x 10^{-1}
centi-	c	x 10^{-2}
milli-	m	x 10^{-3}
micro-	μ (mu) or u	x 10^{-6}
nano-	n	x 10^{-9}
pico-	p	x 10^{-12}
femto-	f	x 10^{-15}

1. 1 METER ≡ 10 deciMETERS
   ≡ 100 centiMETERS
   ≡ 1000 milliMETERS

   1,000 METERS ≡ 1 kiloMETER
2. **1 milli**METER ≡ **1 mm** = 10^{-3} METER

   1 centiMETER ≡ **1 cm** = 10^{-2} METER

   1 deciMETER ≡ **1 dm** = 10^{-1} METER

   1 kiloMETER ≡ **1 km** = 10^{3} METER
3. Any unit can be substituted for METER above.
4. 1 cm^3 ≡ 1 mL ≡ 1 cc
5. 1 liter ≡ 1000 mL ≡ 1 dm^3
6. 1 cm^3 $H_2O_{(l)}$ ≡ 1 mL $H_2O_{(l)}$ = 1.00 g $H_2O_{(l)}$
7. meter = m ; gram = g ; second = s
8. If **prefix- = 10^a**, 1 ***unit*** = **10^{-a}** *prefix*-units
9. To change a prefix definition from a "1 prefix-unit = " format to a "1 base unit = " format, change the exponent sign.
10. **Rules for units in calculations.**
    a. When adding or subtracting, the *units must* be the *same* in the numbers being added and subtracted, and those same units must be added to the answer.
    b. When multiplying and dividing units, the units multiply and divide.
    c. When multiplying and dividing, *group* the numbers, exponentials, and units separately. Solve the three parts, then recombine the terms.

# # # # #

Module 3 – Significant Figures

Pretest: If you think you know how to use significant figures correctly, take the following pretest to be sure. Check your answers at bottom of this *page*. If you do *all* of the pretest perfectly, skip Module 3.

1. How many significant figures are in each of these?

   a. 0.002030 b. 670.0 c. 670 d. 2 (exactly)

2. Round these numbers as indicated.

   a. 62.75 to the tenths place. b. 0.090852 to 3 *sf*.

3. Use a calculator, then express your answer as a number with proper significant figures and units attached.

   $$4.701 \times 10^3 \ \frac{L^2 \bullet g}{s^2} \bullet 0.0401\ s^{-2} \bullet \frac{23.060\ \ s^4}{6.0 \times 10^{-5}\ L} \bullet \text{(an exact 4)} =$$

4. Solve with*out* a calculator. Write your answer in scientific notation with proper units and significant figures. $(56 \times 10^{-10}\ \text{cm}) - (49.6 \times 10^{-11}\ \text{cm}) =$

* * * * *

Lesson 3A: Rules for Significant Figures

Nearly all measurements have *uncertainty*. In science, we need to express

- how much uncertainty exists in measurements, and
- the uncertainty in calculations based on measurements.

The *differentials* studied in calculus provide one method to find a precise range of the uncertainty in calculations based on measurements, but differentials can be time-consuming.

An easier method for expressing uncertainty is **significant figures**, abbreviated in these lessons as *sf***.**

Other methods measure uncertainty more accurately, but significant figures provide a approximation of uncertainty that, compared to other methods, is easy to use in calculations. In first-year chemistry, significant figures in nearly all cases will be the method of choice to indicate an approximation of the uncertainty in measurements and calculations.

* * * * *

Pretest Answers: Your answers must match these exactly.

1a. 4 1b. 4 1c. 2 1d. Infinite *sf* 2a. 62.8 2b. 0.0909 3. 2.9×10^8 L • g

4. 5.1×10^{-9} cm

* * * * *

Significant Figures: Fundamentals

Use these rules when recording measurements and rounding calculations.

1. **To Record a Measurement**

Write all the digits you are sure of, plus the *first* digit that you must *estimate* in the measurement: the first **doubtful digit** (the first **uncertain digit**). Then *stop*.

> When writing a measurement using significant figures, the *last* digit is the first *doubtful* digit. Round measurements to the highest place with doubt.

Example:

If a scale reads mass to the thousandths place, but under the conditions of the experiment the uncertainty in the measurement is ± 0.02 grams, we can write

12.432 g ± 0.02 g using **plus-minus notation** to record uncertainty.

However, in a calculation, if we need to multiply or divide by that measured value, the math to include the ± can be time-consuming. So, to convert the measurement to *significant figure*s notation, we write

12.43 g

When using significant figures to indicate uncertainty, the last place written in a measurement is the first place with doubt. The ± showed that the highest *place* with doubt is the hundredths place. To convert to significant figures, we *round* the recorded digits back to that place, then remove the ± .

We convert the measurement to significant figures notation because in calculations, the math when using numbers like 12.43 follows familiar rules.

2. **To Add and Subtract Using Significant Figures**

a. First, add or subtract as you normally would.

b. Next, search the numbers for the doubtful digit in the *highest place*. The answer's *doubtful* digit must be **in** that *place*. *Round* the answer to that *place*.

Example:

```
      23.1      ←
  +   16.01
  +    1.008
      40.118    =  40.1
```

This answer must be rounded to **40.1** because the tenths place has doubt.

The tenths is the highest *place* with doubt among the numbers added.

Recall that the tenths place is *higher* than the hundredths place, which is higher than the thousandths place.

c. <u>The logic</u>: If you add a number with doubt in the tenths place to a number with doubt in the hundredths place, the answer has doubt in the tenths place.

A doubtful digit is significant, but numbers after it are not.

In a measurement, if the number in a given place is doubtful, numbers after that place are garbage. We allow one *doubtful* digit in answers, but no garbage.

d. <u>Another way to state this rule</u>: When adding or subtracting, round your answer back to the last *full column* on the right. This will be the first column of numbers, moving right to left (←), with no *blanks* above.

The blank space *after* a doubtful digit indicates that we have no idea what that number is, so we cannot add a blank space and get a significant number in the answer in that column.

e. <u>When adding or subtracting using a calculator</u>: Underline the highest place with doubt in the numbers being added and subtracted. Round your calculator answer to that *place*.

Using a calculator, apply the rule to: **Q.** $43 + 1.00 - 2.008 =$

* * * * *

A. $4\underline{\mathbf{3}} + 1.00 - 2.008 = 4\underline{\mathbf{1}}.992$ on the calculator = $\mathbf{42}$ in significant figures

Among the numbers being added and subtracted, the highest doubt is in the one's place. In chemistry calculations, you must round your final answer to that place.

f. When adding and subtracting exponential notation (see Lesson 1B), first make the exponential terms the same, *then* apply the rules above to the significand of the answer.

This rule is in agreement with the general rule: when adding and subtracting, round to the highest *place* with doubt.

<u>Example</u>:

$$\begin{array}{rl} 2.8 \times 10^{-8} & = \quad 2.\underline{8} \quad\;\; \times 10^{-8} \\ -\ 134 \times 10^{-11} & \quad -\ 0.134 \times 10^{-8} \\ & \quad\;\;\; 2.\underline{6}66 \;\; \times 10^{-8} = \mathbf{2.7 \times 10^{-8}} \end{array}$$

<u>Summary:</u> When **adding or subtracting**, round your final answer back to

- the highest *place* with doubt, which is also

When adding or subtracting in columns, this is also

- the *leftmost place* with doubt, which is also
- the last full column on the right, which is also
- the last column to the right with*out* a blank space.

Practice A: First memorize the rules above. *Then* do the problems. When finished, check your answers at the end of the lesson.

1. Convert these from *plus-minus* notation to significant figures notation.

   a. 65.316 mL ± 0.05 mL   b. 5.2 cm ± 0.1 cm

   c. 1.8642 km ± 0.22 km   d. 16.8 ºC ± 1 ºC

2. Add and subtract, with our without calculator. Round your final answer to the proper number of significant figures.

   a. 23.1 + 23.1 + 16.01

   b. 2.016 + 32.18 + 64.5 =

   c. $1.976 \times 10^{-13} - 7.3 \times 10^{-14}$

3. Use a calculator. Round your final answer to the proper number of significant figures.

   a. 2.016 + 32.18 + 64.5 =

   b. 16.00 – 4.034 – 1.008 =

---

3. **To Count Significant Figures**

When multiplying and dividing, we need to *count* the number of significant figures in a measurement. To count the number of *sf*, count the sure digits *plus* the doubtful digit. The doubtful digit is significant.

This rule means that for numbers in a measurement that do not include zeros, the *count* of *sf* in a measurement is simply the number of digits shown.

Examples: 123 meters has **3** *sf*. 14.27 grams has **4** *sf*.

In exponential notation, to find the number of *sf*, look only at the significand. The exponential term does not affect the number of significant figures.

Example: 2.99×10^{8} meters/second has **3** *sf*.

4. **To Multiply and Divide**

This is the rule we will use most often.

a. First multiply or divide as you normally would.

b. Then *count* the *number* of *sf* in *each* of the numbers you are multiplying or dividing.

c. Your answer can have *no more sf* than the measurement with the *least sf* that you multiplied or divided by. *Round* the answer back to that *number* of *sf*.

Example: 3.1865 cm x **8.8** cm = 28.041 = **28** cm^2 (must *round* to **2** *sf*)
^5 *sf*   ^2 *sf*   ^2 *sf*

> **Summary: Multiplying and Dividing**
>
> If you **multiply** and/or **divide** a 10-*sf* number and a 9-*sf* number and a 2-*sf* number, you must round your answer to 2 *sf*.

5. **When Moving the Decimal:** do not change the *number* of *sf* in a significand.

   Q. Convert 424.7×10^{-11} to scientific notation. **A.** 4.247×10^{-9}

6. **In Calculations With Steps or Parts**

   The rules for *sf* should be applied at the **end** of a calculation.

   In problems that have several separate parts (1a, 1b, etc.), and earlier answers are used for later parts, many instructors prefer that you carry one extra *sf* until the end of a calculation, then round to proper *sf* at the final step. This method minimizes changes in the final doubtful digit due to rounding in the steps.

Practice B: First memorize the rules above. *Then* do the problems. When finished, check your answers at the end of the lesson.

1. Multiply and divide using a calculator. Write the first six digits of the calculator result, then write the final answer, with units, and with the proper number of *sf*.

   a. 3.42 cm *times* 2.3 cm^2 =

   b. 74.3 L^2 divided by 12.4 L =

2. Convert to scientific notation: a. 0.0060×10^{-15} b. $1{,}027 \times 10^{-1}$

3. a. 9.76573 **x** 1.3 = A = b. A/2.5 =

ANSWERS: Your answers must match these exactly.

Practice A

1a. 65.316 mL ± 0.05 mL **65.32 mL** The highest *place* with doubt is hundredths.

When converting to *sf*, write all the sure digits. At the first place with doubt, round and stop.

1b. 5.2 cm ± 0.1 cm **5.2 cm** Highest doubt is in tenth's place. Round to tenths.

1c. 1.8642 km ± 0.22 km **1.9 km** The *highest* doubt is in tenth's place. Round to back tenths.

1d. 16.8 °C ± 1 °C **17 °C** Doubt in the one's place. Round back to the highest place with doubt.

2. (a)

   23.1
   + 23.1
   <u>16.01</u>
   62.21 Must round to **62.2**

   (b) 2.016 + 32.18 + 64.<u>5</u> = 98.<u>6</u>96 = Must round to **98.7**

   (c)

   1.976×10^{-13}
   $-$ **<u>0.73</u>** $\times 10^{-13}$
   1.2<u>4</u>6 $\times 10^{-13}$
   Must round to **1.25** $\times 10^{-13}$

3a. 2.016 + 32.18 + 64.**<u>5</u>** = 98.**<u>6</u>**96 - Must round to **98.7**

3b. 16.0**<u>0</u>** — 4.034 —1.008 = 10.9**<u>5</u>**8 Must round to **10.96**

Practice B

1a. **7.9 cm^3 (2 sf)** 1b. **5.99 L (3 sf)** 2a. **6.0 x 10^{-18}** 2b. **1.027 x 10^2**

3a. **12.7** If this answer were not used in part b, the proper answer would be 13 (2 *sf*), but since we need the answer in part b, it is often preferred to carry an extra *sf*. 3b. 12.7/2.5 = **5.1**

* * * * *

Lesson 3B: Special Cases

When using significant figures to express uncertainty, there are special rules for zeros, and exact numbers, and rounding off a 5.

1. **Rounding.** If the number *beyond* the place you are rounding to is
   a. *Less* than 5: Drop it (round *down*). Example: 1.$\underline{3}$42 rounded to *tenths* = 1.3
   b. *Greater* than 5: Round *up*. Example: 1.7$\underline{4}$8 = 1.75 (rounded to underlined place)
   c. A **5** followed by *any non-zero* digits: Round *up*. Example: 1.0$\underline{2}$502 = 1.03

2. **Rounding a lone 5**

   A lone 5 is a 5 without following digits *or* a 5 followed by zeros.

   To round off a lone 5, some instructors prefer the simple "round 5 up" rule. Others prefer a slightly more precise "engineer's rule" described as follows.

   a. If the number in *front* of a lone 5 being rounded off is *even*, round *down* by dropping the 5.

      Example: 1.$\underline{4}$5 = 1.4

   b. If the number in *front* of a lone 5 is *odd*, round it *up*.

      Example: 1.$\underline{3}$500 = 1.4

   A 5 followed by one or more zeros is rounded in the same way as a "lone 5."

   Rounding a lone 5, the rule is "even in front of 5, leave it. Odd? Round up."

   Why not always round 5 up? On a number line, a 5 is exactly halfway between 0 and 10. If you always round 5 up in a large number of calculations, your average will be slightly high. When sending a rover on a 300 million mile trajectory to Mars, if you calculate *slightly* high, you may miss your target by thousands of miles.

   The "even leave it, odd up" rule rounds a lone 5 down half the time and up half the time. This keeps the average of rounding 5 in the middle, where it should be.

   Rounding off a lone 5 or a 0.$\underline{1}$500 is not a case that occurs often in calculations, but when it does, use the rounding rule preferred by the instructor in *your* course.

Practice A: Round to the underlined *place*. Check answers at the end of this lesson.

1. 0.002$\underline{1}$2 2. 0.0$\underline{9}$94 3. 20.0$\underline{5}$61

4. 23.$\underline{2}$5 5. 0.06$\underline{5}$500 6. 0.0$\underline{7}$50 7. 2.$\underline{6}$59 x 10^{-3}

---

3. **<u>Zeros</u>.** When do zeros *count* as *sf*? There are four cases.

   a. *Leading* zeros (zeros in *front* of *all* other digits) are *never* significant.

      <u>Example</u>: 0.0006 has one *sf*. . (Zeros in front never count.)

   b. Zeros embedded *between* other digits *are* always significant.

      <u>Example</u>: 300.07 has 5 *sf*. (Zeros *sandwiched* by *sf count*.)

   c. Zeros *after all other* digits *as well as after* the decimal point *are* significant.

      <u>Example</u>: 565.0 has 4 *sf*. You would not need to include that zero if it were not significant.

   d. Zeros *after* all other digits but *before* the decimal are assumed to be *not* significant.

      <u>Example</u>: 300 is assumed to have 1 *sf*, meaning "give or take at least 100."

   When a number is written as 300, or 250, it is not *clear* whether the zeros are significant. Many science textbooks address this problem by using this rule:

   - "500 meters" means *one sf*, but
   - "500**.** meters," with an *unneeded decimal point* added after a zero, means 3 *sf*.

   These modules will use that convention on occasion as well.

   However, the best way to avoid ambiguity in the number of significant figures is to write numbers in scientific notation.

   4 x 10^2 has *one sf*; **4.00** x 10^2 has *3 sf*.

   In *exponential* notation, only the significand contains the significant figures.

   In *scientific* notation, all of the digits in the significan***d*** are significan***t*** .

* * * * *

Why are zeros complicated? Zero has multiple uses in our numbering system.

In cases 3a and 3d above, the zeros are simply "indicating the place for the decimal." In that role, they are *not* significant as measurements. In the other two cases, the zeros represent numeric values. When the zero represents "a number between a 9 and a 1 in a measurement," it is significant.

4. **<u>Exact numbers</u>**. Measurements with *no* uncertainty have an *infinite* number of *sf*. Exact numbers do not add uncertainty to calculations.

   - If you multiply a 3 *sf* number by an *exact* number, round your answer to 3 *sf*.

   This rule means that *exact* numbers are *ignored* when deciding the *sf* in an calculated answer. In chemistry, we use this rule in situations including the following.

   a. Numbers in *definitions* are exact.

      <u>Example</u>: The relationship "1 km = 1000 meters," is a definition of *kilo-* and not a measurement with uncertainty. Both the 1 and the 1000 are exact numbers. Multiplying or dividing by those *exact* numbers will not limit the number of *sf* in your answer.

b. The number **1** in nearly all cases is *exact*.

Example: The conversion "**1** km = 0.62 miles" is a legitimate approximation, but it is not a *definition* (≡) and is not *exactly* correct. The **1** is therefore assumed to be exact, but the 0.62 has uncertainty and has 2 *sf*.

c. Whole numbers (such as 2 or 6), *if* they are a measure of exact quantities (such as 2 people or 6 molecules), are also exact numbers with infinite *sf*.

d. *Coefficients* and *subscripts* in chemical formulas and equations are exact.

Example: $\mathbf{2}\ H_{\mathbf{2}} + \mathbf{1}\ O_{\mathbf{2}} \rightarrow \mathbf{2}\ H_{\mathbf{2}}O$ All of those *numbers* are exact.

You will be reminded about these exact-number cases as we encounter them. For now, simply remember that exact numbers

- have infinite *sf*, and
- do not limit the *sf* in an answer.

Practice B

Write the number of *sf* in these.

1. 0.0075 2. 600.3 3. 178.40 4. 4640. 5. 800

6. 2.06×10^{-9} 7. 0.060×10^{3} 8. 0.02090×10^{5} 9. 3 (exact)

ANSWERS

Practice A

1. 0.00212 rounds to 0.0021 2. 0.0994 rounds to 0.10 3. 20.0561 rounds to 20.06
4. 23.25 rounds to **23.2**. by the engineer's "lone 5: even, leave it" rule, *or* **23.3** by the "round lone 5 up" rule.
5. 0.065500 rounds to **0.066** by both rules. Eng: the lone 5 to be rounded follows an odd 5. Round "odd up."
6. 0.0750 rounds to **0.08** by both rules. Engineers: When rounding a lone 5, use "even leave it, odd up."
7. 2.659×10^{-3} rounds to $\mathbf{2.7 \times 10^{-3}}$ By all rules: when rounding a 5 followed by *non-zeros*, round up.

Practice B

1. 0.0075 has **2** *sf*. (Zeros in front never count.) 2. 600.3 has **4** *sf*. (Sandwiched zeros count.)
3. 178.40 has **5** *sf*. (Zeros after the decimal and after all the numbers count.)
4. 4640. has **4** *sf*. (Zeros after the numbers but before a written decimal count.)
5. 800 has **1** *sf*. (Zeros after all numbers but before the decimal place usually don't count.)
6. 2.06×10^{-9} has **3** *sf*. (The significand in front contains and determines the *sf*.)
7. 0.060×10^{3} has **2** *sf*. (The significand contains the *sf*. Leading zeros never count.)
8. 0.02090×10^{5} has **4** *sf*. (The significand contains the *sf*. Leading zeros never count. The rest here do.)
9. 3 (exact) **Infinite *sf.*** Exact numbers have no uncertainty and infinite *sf.*

* * * * *

Lesson 3C: Summary and Practice

First, memorize the rules.

1. When expressing a measuring in significant figures, *include* the *first doubtful* digit, then stop. Round measurements to the doubtful digit's place.
2. When counting significant figures, include the doubtful digit.
3. When adding and subtracting *sf*,
   a. find the measurement that has *doubt* in the *highest place.*
   b. *Round* your answer to that *place*.
4. When multiplying and dividing,
   a. find the number in the calculation that has the least number of *sf*.
   b. Round your answer to that number of *sf*.
5. In exponential notation, the *sf* are in the significand.
6. When moving a decimal, keep the same number of *sf* in the significand.
7. When solving a problem with parts, carry an extra *sf* until the final step.
8. To round off a lone 5, use the rule preferred by your instructor. Either always round up, or use "even in front of 5, leave it. Odd? Round *up*."
9. For zeros,
   a. zeros in front of all other numbers are never significant.
   b. Sandwiched zeros are always significant.
   c. Zeros after the other numbers and after the decimal are significant.
   d. Zeros after all numbers but before the decimal place are not significant, but if an unneeded decimal point is shown after a zero, that zero is significant.
10. Exact numbers have infinite *sf*.

For reinforcement, make the flashcards you need using the method in Lesson 2C.

Front-side (with notch at top right):	Back Side -- Answers
Writing measurements in *sf*, stop where?	At the first doubtful digit
Counting the number of *sf*, which digits count?	All the sure, plus the doubtful digit
Adding and subtracting, round to where?	The *column* with doubt in highest place = last full column
Multiplying and dividing, round how?	Least # of *sf* in calculation = # *sf* allowed
In counting *sf*, zeros in front	Never count
Sandwiched zeros	count
Zeros after numbers and after decimal	count
Zeros after numbers but before decimal	Probably don't count

Zeros followed by un-needed decimal	count
Exact numbers have	Infinite *sf*

Run the flashcards until perfect, then start the problems below.

Practice: Try every *other* problem on day 1. Try the rest on day 2 of your practice.

1. Write the number of *sf* in these.

a. 107.42 b. 10.04 c. 13.40 d. 0.00640 e. 0.043×10^{-4}

f. 1590.0 g. 320×10^{9} h. 14 (exact) i. 2500 j. 4200.

2. Round to the place indicated.

a. 5.15 cm (tenths place)
b. 31.84 meters (3 *sf*)
c. 0.819 mL (hundredths place)
d. 0.0635 cm^2 (2 *sf*)
e. 0.04070 g (2 *sf*)
f. 6.255 cm (tenths place)

3. Addition and Subtraction: Round answers to proper *sf*.

a.
$$\begin{array}{r} 1.008 \\ +\ 1.008 \\ \underline{32.00} \end{array}$$

b.
$$\begin{array}{r} 17.65 \\ \underline{-\ \ 9.7} \end{array}$$

c. 39.1 + 124.0 + 14.05 =

4. Multiplication and Division: Write the first 6 digits given by your calculator. Then write the answer with the proper number of *sf* and proper units.

a. 13.8612 cm x 2.02 cm =
b. 4.4 meters x 8.312 meters^2 =
c. 2.03 cm^2/1.2 cm =
d. 0.5223 cm^3/0.040 cm =

5. Use a calculator. Answer in scientific notation with proper *sf*.

a. $(2.25 \times 10^{-2})(6.0 \times 10^{23})$
b. $(6.022 \times 10^{23})\ /\ (1.50 \times 10^{-2})$

6. Convert these from ± to *sf* notation.

a. 2.0646 m ± 0.050 m
b. 5.04 nm ± 0.12 nm
c. 12.675 g ± 0.20 g
d. 24.81 ºC ± 1.0 ºC

Answer 7 and 8 in scientific notation, with proper units and *sf*:

7. $5.60 \times 10^{-2}\ \dfrac{L^2 \bullet g}{s} \bullet 0.090\ s^{-3} \bullet \dfrac{4.00\ \ s^4}{6.02 \times 10^{-5}\ L^3} \bullet$ (an exact 2) =

8. Without a calculator:

$(-\,50.0 \times 10^{-14}\ g) - (-\,49.6 \times 10^{-12}\ g)$ =

9. For additional practice, solve the problems in the *pretest* at the beginning of this *module*.

ANSWERS

1. a. 107.42 **5** *sf* (Sandwiched zeros count.) b. 10.04 **4** (Sandwiched zeros count.)
   c. 13.40 **4** (Zeros after numbers and after the decimal count.)
   d. 0.00640 **3** (Zeros in front never count, but zeros both after #s and after the decimal count.)
   e. 0.043×10^{-4} **2** (Zeros in front never count. The significand contains and determines the *sf*.)
   f. 1590.0 **5** (The last 0 counts since after #s and after decimal. This sandwiches the first 0.)
   g. 320×10^{9} **2?** (Zeros after numbers but before the decimal usually don't count.)
   h. 14 (exact) **Infinite** (Exact numbers have infinite *sf*.)
   i. 2500 **2** (Zeros at the end before the decimal usually don't count.)
   j. 4200. **4** (The *decimal* at the end means the 0 before it counts, and first 0 is sandwiched.)
2. a. 5.15 cm (tenths place) **5.2 cm** (Round up by both lone 5 rules. Eng: 1 is odd, round 5 up.)
   b. 31.84 meters (3 *sf*) **31.8 meters** (3rd digit is last digit: rounding off a 4, round down.)
   c. 0.819 mL (hundredths place) **0.82 mL** (9 rounds up.)
   d. 0.0635 cm^2 (2 *sf*) **0.064 cm^2** (Leading zeros never count.. Round to 2nd *sf*, up by both rules.)
   e 0.04070 g (2 *sf*) **0.041 grams** (Zeros in front never count.)
   f. 6.<u>2</u>55 cm (tenths place) **6.3 cm** (Rounding a 5 *followed* by other digits, always round up.)
3. a.

   ```
     1.008
   + 1.008
    32.00
   -------
    34.016  = 34.02
   ```

   b.

   ```
    17.65
   -  9.7
   -------
     7.95 = 8.0 (5 up or 9=odd up)
   ```

   c.

   ```
     39.1
   + 124.0
     14.05
   -------
    177.15  = 177.2
   ```

4. For help with unit math, see Lesson 2B. For help with exponential math, see Module 1.
   a. 13.8612 cm x 2.02 cm = 27.9996 = **28.0 cm^2 (3 sf)**
   b. 4.4 meters x 8.312 $meters^2$ = 36.5728 = **37 $meter^3$ (2 sf,** 5 plus other digits, always round up)
   c. 2.03 cm^2/1. 2 cm = 1.69166 = **1.7 cm (2 sf)**
   d. 0.5223 cm^3/0.040 cm = 13.0575 = **13 cm^2 (2 sf)**
5. a. $(2.25 \times 10^{-2})(6.0 \times 10^{23})$ = 13.5×10^{21} = **1.4×10^{22}** in scientific notation **(2 sf)**
   b. $(6.022 \times 10^{23}) / (1.50 \times 10^{-2})$ = **4.01×10^{25} (3 sf)**
6. a. 2.0646 m ± 0.050 m **2.06 m** The *highest* doubt is in the hundredth's place. Round to that place.
   b. 5.04 nm ± 0.12 nm **5.0 nm** The highest doubt is in the tenth's place. Round to that place.
   c. 12.675 g ± 0.20 g **12.7 g** The highest doubt is in the tenth's place. Round to that place.
   d. 24.81 ºC ± 1.0 ºC **25 ºC** The highest doubt is in the one's place. Round to that place.

7. $= \dfrac{5.60 \cdot 0.090 \cdot 4.00 \cdot (\text{an exact 2})}{6.02} \cdot \dfrac{10^{-2}}{10^{-5}} \cdot \dfrac{L^2 \cdot g \cdot s^{-3} \cdot s^4}{s \cdot L^3} = 0.67 \times 10^3 = \mathbf{6.7 \times 10^2}\ \dfrac{\mathbf{g}}{\mathbf{L}}$

   The 0.090 limits the answer to 2 *sf*. Exact numbers do not affect *sf*. For unit cancellation, see Lesson 2D. Group and handle numbers, exponentials, and units *separately*.

8. $(-50.0 \times 10^{-14}\ g) - (-49.6 \times 10^{-12}\ g) =$

$$\begin{array}{rl} +\mathbf{49.\underline{6}} & \mathbf{\times 10^{-12}\ g} \\ \underline{\mathbf{-0.500}} & \mathbf{\times \underline{10}^{-12}\ g} \\ \mathbf{49.\underline{1}} & \mathbf{\times 10^{-12}\ g} \quad = \quad \mathbf{4.9\underline{1} \times 10^{-11}\ g} \end{array}$$

   Numbers added or subtracted must have same exponents and units (see Lessons 1B, 2B). Adjusting to the *highest* exponent in the series (—12 is higher than —14) often helps with *sf*. In moving the decimal point, do not change the number of *sf*. Apply the rules for *sf* rounding at the *end* of a calculation.

* * * * *

Lesson 3D: The Atoms – Part 2

To continue to learn the atoms encountered most often, **your assignment is:**

- For the **20** atoms below, memorize the name, symbol, and position in this table. For each atom, given its symbol or name, be able to write the other.
- Be able to fill in an empty table of this shape with those names and symbols in their proper places. (The "atomic numbers" shown above each symbol are optional.)

Periodic Table

1A	2A	3A	4A	5A	6A	7A	8A
1 H Hydrogen							2 He Helium
3 Li Lithium	4 Be Beryllium	5 B Boron	6 C Carbon	7 N Nitrogen	8 O Oxygen	9 F Fluorine	10 Ne Neon
11 Na Sodium	12 Mg Magnesium	13 Al Aluminum	14 Si Silicon	15 P Phosphorus	16 S Sulfur	17 Cl Chlorine	18 Ar Argon
19 K Potassium	20 Ca Calcium						

# # # # #

Module 4 – Conversion Factors

Prerequisites: Module 4 requires knowledge of exponential math and metric fundamentals in Lessons 1A, 1B, 2A, 2B, 3A and 3B. The other lessons in Modules 1-3 will be helpful, but not essential, for Module 4.

Pretests: If the use of conversion factors is easy review, try the *last two problems* in each *lesson*. If you get those right, skip the lesson. If they are not easy, complete the lesson.

* * * * *

Lesson 4A: Conversion Factor Basics

Conversion factors can be used to change from one unit of measure to another, or to find equivalent measurements of substances or processes. A conversion factor is a *ratio* (a *fraction*) made from two measured quantities that are *equal* or *equivalent* in a problem. A conversion factor is a fraction that equals *one*.

Conversion factors have a value of unity (1) because they are made from equalities. For any fraction in which the top and bottom are equal, its value is *one*.

For example: $\frac{7}{7} = 1$

Or, since 1 milliliter = 10^{-3} liters ; $\frac{10^{-3}\text{ L}}{1\text{ mL}} = 1$ and $\frac{1\text{ mL}}{10^{-3}\text{ L}} = 1$

These last two fractions are typical conversion factors. Any fraction that equals *one* right-side up will also equal *one* up-side down. Any conversion factor can be inverted (flipped over) for use if necessary, and it will still equal one.

When converting between liters and milliliters, *all* of these are legal conversion factors:

$\frac{1\text{ mL}}{10^{-3}\text{ L}}$ $\frac{1000\text{ mL}}{1\text{ L}}$ $\frac{10^3\text{ mL}}{1\text{ L}}$ $\frac{3{,}000\text{ mL}}{3\text{ L}}$ All are equal to *one*.

All of those fractions are mathematically equivalent because they all represent the same *ratio*. Upside down, each fraction is also legitimate conversion factor, because the top and bottom are equal, and its value is one.

In solving calculations, the conversions that are *preferred* if available are those that are made from fundamental definitions, such as "milli- = 10^{-3}." However, each of the four forms above is legal to use in converting between milliliters and liters, and either of the first three forms may be encountered during calculations solved in science textbooks.

If a *series* of terms are equal, any *two* of those terms can be used as a conversion factor.

Example: Since 1 meter = 10 decimeters = 100 centimeters = 1000 millimeters

Then each of the following (and others) is a legitimate conversion factor:

$\frac{1000\text{ mm}}{1\text{ m}}$ $\frac{1\text{ mm}}{10^{-3}\text{ m}}$ $\frac{10^2\text{ cm}}{1\text{ m}}$ $\frac{100\text{ cm}}{10\text{ dm}}$ $\frac{10\text{ cm}}{1\text{ dm}}$

Let's try an example of conversion-factor math. Try the following problem. Show your work on this page or in your problem notebook, then check your answer below.

<u>Multiply</u> 7.5 kilometers • $\frac{10^3 \text{ meters}}{1 \text{ kilometer}}$ =

* * * * * (* * * mean: cover below, write your answer, *then* check below.)

<u>Answer</u>

$$7.5 \ \cancel{\text{kilometers}} \bullet \frac{10^3 \text{ meters}}{1 \ \cancel{\text{kilometer}}} = \frac{(7.5 \bullet 10^3)}{1} \text{ meters} = 7.5 \times 10^3 \textbf{ meters}$$

When these terms are multiplied, the "like units" on the top and bottom cancel, leaving meters as the unit on top.

Since the conversion factor multiplies the *given* quantity by *one*, the answer equals the *given* amount that we started with. This answer means that 7,500 meters is the same as 7.5 km.

Multiplying a given quantity by a conversion factor changes the *units* that measure the quantity but does not change its *amount*. The result is what we started with, measured in different units.

This process answers a question posed in many science problems: From the units we are given, how can we obtain the units we want?

Our method of solving calculations will focus on finding equal or equivalent quantities. Using those equalities, we will construct conversion factors to solve problems.

* * * * *

<u>Summary</u>

- Conversion factors are made from two measured quantities that are either defined as equal *or* are equivalent or equal in the problem.
- Conversion factors have a value of one, because the top and bottom terms are equal or equivalent.
- Any *equality* can be made into a conversion (a *fraction* or *ratio)* equal to *one*.
- When the units are set up to cancel correctly, *given* numbers and units multiplied by conversions will result the WANTED numbers and units.

➔ *Units* tell you where to write the numbers to solve a calculation correctly.

To *check* the metric conversion factors encountered most often, use these rules.

Since conversions must be *equal* on the top and bottom, the equalities that define the metric prefixes can be used to write and check conversions.

- 1 ***milli***unit or 1 **m**(unit abbreviation) must be above or below $\mathbf{10^{-3}}$ units ;
- 1 ***centi***- or 1 **c**- must be above or below $\mathbf{10^{-2}}$;
- 1 ***kilo***- or 1 **k**- must be above or below $\mathbf{10^{3}}$.

Practice: Try every other lettered problem. Check your answers frequently. If you miss one on a section, try a few more. Answers are on the next page.

1. Multiply the conversion factors. Cancel units that cancel, then group the numbers and do the math. Write the answer number and unit in scientific notation.

   a. $225 \text{ centigrams} \cdot \frac{10^{-2} \text{ gram}}{1 \text{ centigram}} \cdot \frac{1 \text{ kilogram}}{10^3 \text{ grams}} =$

   b. $1.5 \text{ hours} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} =$

2. To be legal, the top and bottom of conversion factors must be equal. Label these conversion factors as *legal* or *illegal*.

   a. $\frac{1000 \text{ mL}}{1 \text{ liter}}$ b. $\frac{1000 \text{ L}}{1 \text{ mL}}$ c. $\frac{1.00 \text{ g } H_2O}{1 \text{ mL } H_2O}$ d. $\frac{10^{-2} \text{ volt}}{1 \text{ centivolt}}$

   e. $\frac{1 \text{ mL}}{1 \text{ cc}}$ f. $\frac{10^3 \text{ cm}^3}{1 \text{ L}}$ g. $\frac{10^3 \text{ kilowatts}}{1 \text{ watt}}$ h. $\frac{1 \text{ kilocalorie}}{10^3 \text{ calories}}$

3. Place a 1 in front of the unit with a prefix, then complete the conversion factor.

   a. $\frac{\_\_\_\_ \text{ grams}}{\text{kilograms}}$ b. $\frac{\_\_\_\_ \text{ mole}}{\text{nanomole}}$ c. $\frac{\_\_\_\_ \text{ picocurie}}{\text{curie}}$

4. Add numbers to make legal conversion factors, with at least one of the numbers in each conversion factor being a 1.

   a. $\frac{\_\_\_\_ \text{ centijoules}}{\text{joules}}$ b. $\frac{\_\_\_\_ \text{ liters}}{\text{cubic cm}}$ c. $\frac{\_\_\_\_ \text{ cm}^3}{\text{mL}}$

5. Finish these.

   a. $27\text{A} \cdot \frac{2\text{T}}{8\text{A}} \cdot \frac{4\text{W}}{3\text{T}} =$

   b. $2.5 \text{ meters} \cdot \frac{1 \text{ cm}}{10^{-2} \text{ meter}} =$

   c. $\frac{95 \text{ km}}{\text{hour}} \cdot \frac{0.625 \text{ miles}}{1 \text{ km}} =$

   d. $\frac{27 \text{ } meters}{\text{seconds}} \cdot \frac{60 \text{ s}}{1 \text{ min.}} \cdot \frac{1 \text{ kilometer}}{10^3 \text{ } meters} =$

ANSWERS

1 a. $225\ \cancel{\text{centigrams}} \bullet \dfrac{10^{-2}\ \text{gram}}{1\ \cancel{\text{centigram}}} \bullet \dfrac{1\ \textbf{kilogram}}{10^3\ \cancel{\text{grams}}} = \dfrac{225 \times 10^{-2} \times 1}{1 \times 10^3}\ \textbf{kg} = \mathbf{2.25 \times 10^{-3}\ kg}$

The answer means that **2.25×10^{-3}** kg is *equal* to 225 cg.

b. $1.5\ \text{hours} \bullet \dfrac{60\ \text{minutes}}{1\ \text{hour}} \bullet \dfrac{60\ \text{seconds}}{1\ \text{minute}} = \dfrac{1.5 \times 60 \times 60}{1}\ \text{s} = \mathbf{5{,}400\ s}$ *or* $\mathbf{5.4 \times 10^3\ s}$

Recall that **s** is the abbreviation for seconds. This answer means that 1.5 hours is equal to 5,400 s.

2. a. $\dfrac{1000\ \text{mL}}{1\ \text{liter}}$ **Legal**

b. $\dfrac{1000\ \text{L}}{1\ \text{mL}}$ **Illegal**

c. $\dfrac{1.00\ \text{g}\ H_2O}{1\ \text{mL}\ H_2O}$ **Legal IF *liquid* water**

d. $\dfrac{10^{-2}\ \text{volt}}{1\ \text{centivolt}}$ **Legal**

e. $\dfrac{1\ \text{mL}}{1\ \text{cc}}$ **Legal**

f. $\dfrac{10^3\ \text{cm}^3}{1\ \text{L}}$ **Legal**

g. $\dfrac{10^3\ \text{kilowatts}}{1\ \text{watt}}$ **Illegal**

h. $\dfrac{1\ \text{kilocalorie}}{10^3\ \text{calories}}$ **Legal**

3. a. $\dfrac{\mathbf{10^3}\ \text{grams}}{\mathbf{1}\ \text{kilogram}}$ b. $\dfrac{\mathbf{10^{-9}}\ \text{mole}}{\mathbf{1}\ \text{nanomole}}$ c. $\dfrac{\mathbf{1}\ \text{picocurie}}{\mathbf{10^{-12}}\ \text{curie}}$

4. a. $\dfrac{1\ \text{centijoule}}{\mathbf{10^{-2}}\ \text{joules}}$ *or* $\dfrac{\mathbf{100}\ \text{centijoules}}{\mathbf{1}\ \text{joule}}$ b. $\dfrac{\mathbf{1}\ \text{liter}}{\mathbf{1000}\ \text{cc.}}$ *or* $\dfrac{\mathbf{10^{-3}}\ \text{liters}}{\mathbf{1}\ \text{cubic cm}}$ c. $\dfrac{\mathbf{1}\ \text{cm}^3}{\mathbf{1}\ \text{mL}}$

Either fixed decimal numbers (such as 100) or equivalent exponentials (10^2) may be used in conversions.

5. a. $27\text{A} \bullet \dfrac{2\text{T}}{8\text{A}} \bullet \dfrac{4\text{W}}{3\text{T}} = 27\cancel{\text{A}} \bullet \dfrac{2\cancel{\text{T}}}{8\cancel{\text{A}}} \bullet \dfrac{4\text{W}}{3\cancel{\text{T}}} = \dfrac{27 \bullet 2 \bullet 4}{8 \bullet 3} \bullet \textbf{W} = \mathbf{9W}$

b. $2.5\ \cancel{\text{meters}} \bullet \dfrac{1\ \text{cm}}{10^{-2}\ \cancel{\text{meter}}} = 2.5 \times 10^2\ \text{cm} = \mathbf{250\ cm}$

c. $\dfrac{95\ \cancel{\text{km}}}{\text{hour}} \bullet \dfrac{0.625\ \text{miles}}{1\ \cancel{\text{km}}} = \dfrac{95 \bullet 0.625}{1}\ \dfrac{\text{mi}}{\text{hr}} = \mathbf{59}\ \dfrac{\textbf{miles}}{\textbf{hour}}$

d. $\dfrac{27\ \cancel{\text{meters}}}{\cancel{\text{seconds}}} \bullet \dfrac{60\ \cancel{\text{seconds}}}{1\ \text{min}} \bullet \dfrac{1\ \text{kilometer}}{10^3\ \cancel{\text{meters}}} = \dfrac{27 \bullet 60}{10^3}\ \dfrac{\text{km}}{\text{min}} = \mathbf{1.6}\ \dfrac{\textbf{km}}{\textbf{min}}$

* * * * *

Lesson 4B: Single Step Conversions

In the previous lesson, conversion factors were supplied. In this lesson, you will learn to make your own conversion factors to solve problems. Let's learn the method with a simple example.

Q. How many years is 925 days?

In your notebook, write an answer to each step below.

Steps for Solving with Conversion Factors

1. Begin by writing a question mark (?) and then the *unit* you are *looking for* in the problem, the *answer unit*.
2. Next write an equal sign. It means, "OK, that part of the problem is done. From here on, leave the answer unit alone." You don't cancel the answer unit, and you don't multiply by it.
3. After the = sign, write the number and unit you are *given* (the known quantity).

* * * * *

At this point, in your notebook should be **? years = 925 days**

4. Next, write a • and a line _______ for a conversion factor to multiply by.
5. A key step: write the *unit* of the *given* quantity in the denominator (on the bottom) of the conversion factor. Leave room for a number in front.

   Do *not* put the given *number* in the conversion factor -- just the given *unit*.

   $$? \text{ years} = 925 \text{ days} \cdot \frac{\quad}{\textbf{days}}$$

   This step puts the given unit where it must be to cancel and tells you one part of what the next conversion must include.
6. Next, write the answer unit on the top of the conversion factor.

   $$\textbf{? years} = 925 \text{ days} \cdot \frac{\textbf{year}}{\text{days}}$$

7. Add numbers that make the numerator and denominator of the conversion factor *equal*. In a legal conversion factor, the top and bottom quantities must be equal or equivalent.
8. Cancel the units that you set up to cancel.
9. *If* the unit on the right side after cancellation is the answer unit, stop adding conversions. Write an = sign. Multiply the *given* quantity by the conversion factor. Write the number and the un-canceled unit. Done!

Finish the above steps, then check your answer below.

* * * * *

$$? \text{ years} = 925 \cancel{\text{days}} \cdot \frac{\textbf{1} \text{ year}}{\textbf{365} \cancel{\text{days}}} = \frac{925 \text{ years}}{365} = \textbf{2.53 years}$$

(SF: **1** is *exact*, 925 has 3 *sf*, 365 has 3 *sf* (1 yr. = 365.24 days is more precise), round to 3 *sf*.)

You may need to look back at the above steps, but you should not need to memorize them. By doing the following problems, you will quickly learn what you need to know.

Practice: After each numbered problem, check your answers at the end of this lesson. Look back at the steps if needed.

For the problems in this practice section, write conversions in which *one* of the numbers (in the numerator *or* the denominator) is a 1.

If these are easy, do every third letter. If you miss a few, do a few more.

1. Add numbers to make these conversion factors legal, cancel the units that cancel, multiply the *given* by the conversion, and write your answer.

   a. ? days = 96 hours • $\frac{\text{day}}{24\ \text{hours}}$ =

   b. **?** mL = 3.50 liters • $\frac{1\ \text{mL}}{\text{liter}}$ =

2. To start these, put the *unit* of the *given* quantity where it will cancel. Then finish the conversion factor, do the math, and write your answer with its unit.

   a. **?** seconds = 0.25 minutes • $\frac{\text{sec.}}{1}$ =

   b. **?** kilograms = 250 grams • $\frac{\text{kilogram}}{10^3}$ =

   c. **?** days = 2.73 years • 365 ________ =

   d. **?** years = 200. days • 1 ________ =

3. You should not need to memorize the written rules for arranging conversion factors, however, it is helpful to use this "single unit starting template."

   > When solving for single units, begin with
   >
   > **?** unit WANTED = number and UNIT *given* • $\frac{\qquad\qquad}{\textbf{UNIT}\ \textbf{\textit{given}}}$

   The template emphasizes that your first conversion factor puts the given *unit* (but *not* the given *number*) where it will cancel.

   a. **?** months = 5.0 years • ________ =

   b. **?** liters = 350 mL • =

   c. **?** minutes = 5.5 hours

4. Use the starting template to find how many hours equal 390 minutes.

   ?

5. **?** milligrams = 0.85 kg • $\frac{\text{gram}}{\text{kg}}$ • $\frac{\qquad}{\text{gram}}$ =

ANSWERS

Some but not all unit cancellations are shown. For your answer to be correct, it must include its unit.

Your conversions may be in different formats, such as 1 meter = 100 cm *or* 1 cm = 10^{-2} meters , as long as the top and bottom are equal and the answer is the same as below.

1. a. ? days = 96 ~~hours~~ • $\frac{1 \text{ day}}{24 \text{ hours}}$ = $\frac{96}{24}$ days = **4.0 days**

   b. **?** mL = 3.50 ~~liters~~ • $\frac{1 \text{ mL}}{10^{-3} \text{ liter}}$ = 3.50 • 10^3 mL = **3.50 x 10^3 mL**

   (*SF*: 3.50 has 3 *sf*, prefix definitions are exact with infinite *sf*, answer is rounded to 3 *sf*)

2. Your conversions may be different (for example, you may use **1,000 mL = 1 L *or* 1 mL = 10^{-3} L**), but you must get the same answer.

   a. **?** seconds = 0.25 ~~minutes~~ • $\frac{60 \text{ sec.}}{1 \text{ minute}}$ = 0.25 • 60 sec. = **15 s**

   (*SF*: 0.25 has 2 *sf*, 1 min = 60 sec. is a definition with infinite *sf*, answer is rounded to 2 *sf*)

   b. **?** kilograms = 250 grams • $\frac{1 \text{ kilogram}}{10^3 \text{ grams}}$ = $\frac{250}{10^3}$ kg = **0.25 kg**

   c. **?** days = 2.73 years • $\frac{365 \text{ days}}{1 \text{ year}}$ = 2.73 • 365 days = **996 days**

   d. **?** years = 200. days • $\frac{1 \text{ year}}{365 \text{ days}}$ = $\frac{200}{365}$ years = **0.548 years**

3. a. **?** months = 5.0 years • $\frac{12 \text{ months}}{1 \text{ year}}$ = **60. months**

   (*SF*: 5.0 has 2 *sf*, 12 mo. = 1 yr. is a definition with infinite *sf*, round to 2 *sf* , the 60. decimal means 2 *sf*)

   b. **?** liters = 350 mL • $\frac{10^{-3} \text{ liter}}{1 \text{ mL}}$ = 350 x 10^{-3} liters = **0.35 L**

   (m- = milli- = 10^{-3}. *SF*: 350 has 2 *sf*, prefix definitions are *exact* with infinite *sf*, round to 2 *sf*)

   c. **?** minutes = 5.5 hours • $\frac{60 \text{ minutes}}{1 \text{ hour}}$ = **330 minutes**

4. **? hours = 390 minutes • $\frac{\text{1 hour}}{\text{60 minutes}}$ = 6.5 hours**

5. **?** milligrams = 0.85 ~~kg~~ • $\frac{\mathbf{10^3}\ \text{\sout{gram}}}{\mathbf{1}\ \text{\sout{kg}}}$ • $\frac{\text{1 mg}}{\mathbf{10^{-3}}\ \text{\sout{gram}}}$ = 0.85×10^6 mg = **8.5×10^5 mg**

* * * * *

Lesson 4C: Multi-Step Conversions

In Problem 5 at the end of the previous lesson, we did not know a direct conversion from kilograms to milligrams. However, we knew a conversion from kilograms to grams, and another from grams to milligrams.

In most problems, you will not know a single conversion from the *given* to wanted unit, but there will be known conversions that you can *chain together* to solve.

Try this two-step conversion, based on Problem 5 above. Answer in scientific notation.

Q. **?** milliseconds = 0.25 minutes

* * * * *

A. **?** milliseconds = 0.25 minutes • $\frac{\textbf{60 s}}{\textbf{1 min.}}$ • $\frac{\textbf{1 ms}}{\mathbf{10^{-3}}\ \textbf{s}}$ = 15×10^3 ms = **1.5×10^4 ms**

The 0.25 has two *sf*, both conversions are exact definitions that do not affect the significant figures in the answer, so the answer is written with two *sf.*

The rules are, when

Solving With Multiple Conversions

- If the unit on the right after you cancel units is *not* the answer unit, get rid of it. Write it in the next conversion factor where it will cancel.
- Finish the next conversion with a known conversion, one that either *includes* the answer unit, or gets you *closer* to the answer unit.
- In making conversions, set up *units* to cancel, but add *numbers* that make legal conversions.

Practice

Convert your final answers to scientific notation.

These are in pairs. If Part A is easy, go to Part A of the next question. If you need help with Part A, do Part B for more practice.

1. a. **?** gigagrams = 760 milligrams • ________gram • ______________ =

   b. **?** cg = 4.2 kg • ___________• _______________ =
   g

2. a. ? years = 2.63×10^4 hours • ___________ • ____________ =

   b. ? seconds = 1.00 days • __________hr • ________ • __________ =

3. a. ? µg $H_2O_{(l)}$ = 1.5 cc $H_2O_{(l)}$ • ________g $H_2O_{(l)}$ • ____________ =

   b. ? kg $H_2O_{(liquid)}$ = 5.5 liter $H_2O_{(l)}$ • __________ • _____________ • __________ =

ANSWERS

For visibility, not all cancellations are shown, but cancellations should be marked on your paper.

Your conversions may be different (for example, you may use **1,000 mL = 1 L** ***or*** **1 mL = 10^{-3} L**), but you must get the same answer.

1a. **?** gigagrams = 760 milligrams • $\frac{10^{-3}\text{ g}}{1\text{ mg}}$ • $\frac{1\text{ Gg}}{10^{9}\text{ g}}$ = 760×10^{-12} Gg = **7.6×10^{-10} Gg**

b. **?** cg = 4.2 kg • $\frac{10^3\text{ g}}{1\text{ kg}}$ • $\frac{1\text{ cg}}{10^{-2}\text{ g}}$ = **4.2×10^5 cg**

2a. ? years = 2.63×10^4 hours • $\frac{1\text{ day}}{24\text{ hr.}}$ • $\frac{1\text{ yr}}{365\text{ days}}$ = $\frac{2.63 \times 10^4}{24 \cdot 365}$ = **3.00×10^0 years**

b. ? seconds = 1.00 days • $\frac{24\text{ }\cancel{\text{hr}}}{1\text{ day}}$ • $\frac{60\text{ }\cancel{\text{min}}}{1\text{ }\cancel{\text{hr}}}$ • $\frac{60\text{ s}}{1\text{ }\cancel{\text{min}}}$ = **8.64×10^4 s**

3a. ? μg $H_2O_{(l)}$ = 1.5 cc $H_2O_{(l)}$ • $\frac{\mathbf{1.00\ g\ H_2O_{(l)}}}{\mathbf{1\ cc\ H_2O_{(l)}}}$ • $\frac{\mathbf{1\ \mu g}}{\mathbf{10^{-6}\ g}}$ = **1.5 x 10^{6} μg $H_2O_{(l)}$**

b. ? kg $H_2O_{(l)}$ = 5.5 liter $H_2O_{(l)}$ • $\frac{\mathbf{\cancel{1\ mL}}}{\mathbf{10^{-3}\ L}}$ • $\frac{1.00\ g\ H_2O_{(l)}}{\mathbf{1\ \cancel{mL}}\ H_2O_{(l)}}$ • $\frac{\mathbf{1\ kg}}{\mathbf{10^3\ g}}$ = $\frac{5.5}{10^0}$ = **5.5 kg $H_2O_{(l)}$**

* * * * *

Lesson 4D: English/Metric Conversions

Using Familiar Conversions

All of the conversions between units that we have used so far have had the number **1** on either the top or the bottom, but a one is not required in a legal conversion.

Both "1 kilometer = 1,000 meters" and "3 kilometers = 3,000 meters" are true equalities, and both equalities could be used to make legal conversion factors. In most cases, however, conversions with a 1 are preferred.

Why? We want conversions to be *familiar*, so that we can write them automatically and quickly check that they are correct. Definitions are usually based on *one* of one component, such as "1 km = 10^3 meters." Definitions are the most familiar equalities, and are therefore preferred in conversions.

However, some conversions may be familiar even if they do not include a **1.** For example, many cans of soft drinks are labeled "12.0 fluid ounces (355 mL)." This supplies an equality for English-to-metric volume units: 12.0 fluid ounces **=** 355 mL. That is a legal conversion and, because its numbers and units are seen often, it is a good conversion to use because it is easy to remember and check.

Bridge Conversions

Science problems often involve a key *bridge* conversion between one unit system, quantity, or substance, and another.

For example, a *bridge* conversion between metric and English-system *distance* units is

2.54 centimeters ≡ 1 inch

In countries that use English units, this is now the exact *definition* of an inch. Using this equality, we can convert between metric and English measurements of distance.

Any metric-English distance equality can be used to convert between distance measurements in the two systems. Another metric-English conversion for distance that is frequently used (but not exact) is 0.61 mile = 1 km . (When determining the significant figures, for conversions based on equalities that are not exact definitions, assume that an integer **1** is *exact*, but the other number is precise only to the number of *sf* shown.)

In problems that require bridge conversions, our strategy to will be to "head for the bridge," to *begin* by converting to one of the two units in the *bridge* conversion**.**

When a problem needs a bridge conversion, use these steps.

1) First convert the *given* unit to the unit in the *bridge* conversion that is in the *same system* as the *given* unit.
2) Next, multiply by the bridge conversion. The bridge conversion crosses over from the *given* system to the WANTED system.
3) Multiply by other conversions in the WANTED system to get the answer *unit* WANTED.

Conversions between the metric and English systems provide a way to practice the bridge-conversion methods that we will use in chemical reaction calculations. Add these English distance-unit definitions to your list of memorized conversions.

12 inches ≡ 1 foot   3 feet ≡ 1 yard   5,280 feet ≡ 1 mile

Also commit to memory this metric-to-English bridge conversion for distance.

2.54 cm ≡ 1 inch

Then cover the answer below and apply the steps and conversions above to this problem.

Q. **?** feet = 1.00 meter

* * * * *

Answer

Since the *wanted* unit is English, and the *given* unit is metric, an English/metric bridge is needed.

Step 1: Head for the bridge. Since the *given* unit (meters) is metric system, convert to the *metric unit* used in the bridge conversion (2.54 cm = 1 inch) -- *centimeters*.

$$? \text{ feet} = 1.00 \ \cancel{\text{meter}} \bullet \frac{1 \ cm}{10^{-2} \ \cancel{\text{m}}} \bullet \frac{\qquad}{cm}$$

Note the start of the next conversion. Since cm is not the wanted answer unit, cm *must* be put in the next conversion where it will cancel. If you *start* the "next unit to cancel" conversion automatically after finishing the prior conversion, it helps to arrange and choose the next conversion.

Adjust and complete your work if needed.

* * * * *

Step 2: Complete the bridge that converts to the *system* of the answer: English units.

$$? \text{ feet} = 1.00 \ \cancel{\text{meter}} \bullet \frac{1 \ cm}{10^{-2} \ \cancel{\text{m}}} \bullet \frac{\mathbf{1\ inch}}{\mathbf{2.54}\ cm} \bullet \frac{\qquad}{\mathbf{inch}}$$

* * * * *

Step 3: Get rid of the unit you've got. Get the unit you want.

$$? \text{ feet} = 1.00 \ \cancel{\text{meter}} \bullet \frac{1 \ cm}{10^{-2} \ \cancel{\text{m}}} \bullet \frac{\mathbf{1\ inch}}{\mathbf{2.54}\ cm} \bullet \frac{1 \ \mathbf{foot}}{12 \text{ inches}} = \boxed{3.28 \text{ feet}}$$

The answer tells us that 1.00 meter (the *given* quantity) is equal to 3.28 feet.

Some science problems take 10 or more conversions to solve. However, if you know that a bridge conversion is needed, "heading for the bridge" breaks the problem into pieces, which will simplify your navigation to the answer.

Practice: Use the inch-to-centimeter bridge conversion above. Start by doing every other problem. Do more if you need more practice.

1. ? cm = 12.0 inches • __________ =

2. ? inches = 1.00 meters • __________ • __________

3. For ? inches = 760. mm
   a. To what unit to you aim to convert the *given* in the initial conversions? Why?
   b. Solve: ? inches = 760. mm

4. ? mm = 0.500 yards

5. For ? km = 1.00 mile , to convert using 1 inch = 2.54 cm ,
   a. To what unit to you aim to convert the *given* in the *initial* conversions? Why?
   b. Solve: **?** km = 1.00 mile

6. Use as a bridge for metric mass and English weight units, 1 kilogram = 2.2 lbs.

   ? grams = 7.7 lbs

7. Use the "soda can" volume conversion (12.0 fluid ounces = 355 mL).

   ? fl. oz. = 2.00 liters

8. For the following symbols, write the name of the atom.
   a. C = _______________ b. Cl = ______________ c. Ca = _____________

ANSWERS

In these answers, some but not all of the unit cancellations are shown. The definition 1 cm = 10 mm may be used for *mm* to *cm* conversions. Doing so will change the number of conversions but not the answer.

1. ? cm = 12.0 inches • $\frac{\textbf{2.54 cm}}{\textbf{1 inch}}$ = 12.0 • 2.54 = **30.5 cm** (check how many cm are on a 12 inch ruler)

2. ? inches = 1.00 ~~meters~~ • $\frac{\textbf{1 } \not{\textbf{cm}}}{\textbf{10}^{-2}\ \not{\textbf{m}}}$ • $\frac{\textbf{1 inch}}{\textbf{2.54 } \not{\textbf{cm}}}$ = $\frac{1}{2.54}$ x 10^2 = 0.394 x 10^2 in. = **39.4 inches**

3a. Aim to convert the *given* unit (mm) to the one unit in the *bridge* conversion that is in the same system (English or metric) as the *given*. **Cm** is the bridge unit that is in the same measurement *system* as mm.

3b. ? inches = 760. mm • $\frac{10^{-3}\ \text{meter}}{1\ \text{mm}}$ • $\frac{1\ \text{cm}}{\textbf{10}^{-2}\ \textbf{m}}$ • $\frac{\textbf{1 inch}}{\textbf{2.54 cm}}$ = $\frac{760 \times 10^{-1}}{2.54}$ in. = **29.9 inches**

SF: 760., with the *decimal* after the 0, means 3 *sf*. Metric definitions and **1** have infinite *sf*. The answer must be rounded to 3 *sf* (see Module 3).

4. ? mm = 0.500 yd. • $\frac{3\ \text{ft.}}{1\ \text{yd.}}$ • $\frac{12\ \text{in.}}{1\ \text{ft.}}$ • $\frac{\textbf{2.54 cm}}{\textbf{1 inch}}$ • $\frac{10^{-2}\ \text{meter}}{1\ \text{cm}}$ • $\frac{\textbf{1 mm}}{\textbf{10}^{-3}\ \textbf{m}}$ = **457 mm**

5a. Aim to convert the *given* unit (miles) to the bridge unit in the same system (English or metric) as the *given*. **Inches** is in the same system as miles.

5b. ? km = 1.00 *mile* • $\frac{5{,}280\ \text{ft.}}{1\ \textit{mile}}$ • $\frac{12\ \text{in.}}{1\ \text{ft.}}$ • $\frac{\textbf{2.54 cm}}{\textbf{1 inch}}$ • $\frac{10^{-2}\ \text{m}}{1\ \text{cm}}$ • $\frac{1\ \text{km}}{10^{3}\ \text{m}}$ = **1.61 km**

SF: Assume an integer **1** that is part of any equality or conversion is exact, with infinite *sf.*

6. ? grams = 7.7 lbs • $\frac{\textbf{1 kg}}{\textbf{2.2 lb}}$ • $\frac{\textbf{10}^{3}\ \textbf{grams}}{\textbf{1 kg}}$ = **3.5 x 10^3 grams**

SF: 7.7 and 2.2 have 2 *sf*. A 1 and metric-prefix *definitions* have infinite *sf*. Round the answer to 2 *sf*.

7. ? fluid ounces = **2.00** liters • $\frac{\textbf{1 mL}}{\textbf{10}^{-3}\ \textbf{L}}$ • $\frac{\textbf{12.0 fl. oz.}}{\textbf{355 mL}}$ = **67.6 fl. oz.** (Check this answer on any 2-liter soda bottle.)

8a. C = **Carbon** b. Cl = **Chlorine** c. Ca = **Calcium**

* * * * *

Lesson 4E: Ratio Unit Conversions

Long Distance Cancellation

The order in which numbers are multiplied does not affect the result. For example, 1 x 2 x 3 has the same answer as 3 x 2 x 1.

The same is true when multiplying symbols or units. While some sequences may be easier to set up or understand, from a mathematical perspective the order of multiplication does not affect the answer.

The following problem is an example of how units can cancel in separated as well as adjacent conversions. Try

Q1: Multiply. Cancel units that cancel. Write the answer number and its unit.

$$\frac{12\ \textit{meters}}{\textbf{sec.}} \cdot \frac{60\ \textbf{sec.}}{1\ \text{min}} \cdot \frac{60\ \text{min.}}{1\ \text{hour}} \cdot \frac{1\ \text{kilometer}}{1000\ \textit{meters}} \cdot \frac{0.62\ \text{miles}}{1\ \text{kilometer}} =$$

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$$\frac{12\ \cancel{\textit{meters}}}{\cancel{\textbf{sec.}}} \cdot \frac{60\ \cancel{\textbf{sec.}}}{1\ \cancel{\text{min.}}} \cdot \frac{60\ \cancel{\text{min.}}}{1\ \textbf{hour}} \cdot \frac{1\ \cancel{\text{kilometer}}}{1000\ \cancel{\textit{meters}}} \cdot \frac{0.62\ \textbf{miles}}{1\ \cancel{\text{kilometer}}} = \mathbf{27}\ \frac{\textbf{miles}}{\textbf{hr.}}$$

This answer means that a speed of 12 meters/sec is the *same* as 27 miles/hour.

Ratio Units in the Answer

In these lessons, we will use the term *single unit* to describe a unit that has one kind of base unit in the numerator but no denominator (which means the denominator is **1**). Single units measure *amounts*. Meters, grams, minutes, milliliters, and cm^3 are all single units. These base units may have prefixes or powers, but must otherwise be or be equivalent to one unit that measures a fundamental quantity.

We will use the term *ratio unit* to describe a fraction that has *one* base unit in the numerator and *one* base unit in the denominator. If a problem asks you to find

meters per second *or* meters/second *or* $\frac{\text{meters}}{\text{second}}$ *or* $\text{m} \cdot \text{s}^{-1}$

all of those terms are identical, and the problem is asking for a ratio unit. During *conversion* calculations, all ratio units should be written in the *fraction* form with a top and bottom.

In Module 11, we will address in detail the different characteristics of single units and ratio units. For now, the distinctions above will allow us to solve problems.

Converting the Denominator

In solving for single units, we have used a starting template that includes canceling a *given* single unit.

> <u>When solving for single units</u>, begin with
>
> ? unit WANTED = # and UNIT *given* • $\frac{\qquad\qquad}{\textit{UNIT given}}$

When solving for ratio units, we may need to cancel a denominator (bottom) unit to start a problem. To do so, we will loosen our *starting* rule to say this.

<u>When Solving With Conversion Factors</u>

If a unit to the right of the equal sign, in or after the *given*, on the top or the bottom,

- *matches* a unit in the answer unit, in both what it is and where it is, (circle) that unit on the right side and do not convert it further;
- is *not* what you WANT, put it where it will cancel, and convert until it matches what you WANT.

After canceling units, if the unit or units to the right of the equal sign match the answer unit, stop adding conversions, do the math, and write the answer.

Q2: Use the rule above to solve.

$$? \frac{\text{cm}}{\text{min.}} = 0.50 \frac{\text{cm}}{\text{s}} \bullet \underline{\hspace{3cm}} =$$

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<u>Answer</u>

$$? \frac{\text{cm}}{\text{min}} = 0.50 \frac{\text{(cm)}}{\text{s}} \bullet \frac{60 \text{ s}}{1 \text{ (min.)}} = \frac{30. \text{ cm}}{\text{min.}}$$

Start by comparing the *wanted* units to the *given* units.

Since you WANT cm on top, and are *given* cm on top, circle (cm) to say, "The top is done. Leave the top alone."

On the bottom, you have seconds, but you WANT minutes. Put seconds where it will cancel. Convert to minutes on the bottom.

When the units on the right match the units you WANT on the left for the answer, stop conversions and do the math.

<u>Practice A</u>: Do Problem 2. Then do Problem 1 if you need more practice.

1. $$? \frac{\text{g}}{\text{dL}} = 355 \frac{\text{g}}{\text{L}} \bullet \underline{\hspace{3cm}} =$$

2. $$? \frac{\text{meters}}{\text{second}} = \frac{4.2 \times 10^5 \text{ meters}}{\text{hour}} \bullet \underline{\hspace{3cm}} \bullet \underline{\hspace{3cm}} =$$

Converting Both Top and Bottom Units

Many problems require converting both numerator and denominator units. In the following problem, an order to convert both units is specified. Write what must be placed in the blanks to make legal conversions, cancel units, do the math, and write then check your answer below.

Q3. $? \frac{\text{meters}}{\text{s}} = 740 \frac{\text{cm}}{\text{min.}} \cdot \frac{\_\_\_\_\_\_}{\text{cm}} \cdot \frac{\text{minutes}}{\_\_\_\_\_\_} =$

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Answer

$? \frac{\text{meters}}{\text{s}} = 740 \frac{\text{cm}}{\text{min}} \cdot \frac{10^{-2}\ \text{meters}}{1\ \text{cm}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 0.12 \frac{\text{meters}}{\text{s}}$

In the *given* on the right, cm is *not* the unit WANTED on top, so put it where it will cancel, and convert to the unit you want on top.

Next, since minutes are on the bottom on the right, but seconds are WANTED, put minutes where it will cancel. Convert to the seconds WANTED.

When chaining conversions, which unit you convert first — the top or bottom unit — makes no difference. The order in which you multiply factors does not change the answer.

On the following problem, no order for the conversions is specified. Add legal conversions in any order, solve, then check your answer below. Before doing the math, double check each conversion, one at a time, to make sure it is legal.

Q4. $? \frac{\text{centigrams}}{\text{liter}} = 0.550 \times 10^{-2} \frac{\text{g}}{\text{mL}}$

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Answer: Your conversions may be in a different order.

$? \frac{\text{centigrams}}{\text{liter}} = 0.550 \times 10^{-2} \frac{\text{g}}{mL} \cdot \frac{1\ \text{cg}}{10^{-2}\ \text{g}} \cdot \frac{1\ mL}{10^{-3}\ \text{L}} = \mathbf{5.50 \times 10^{2}\ \frac{cg}{L}}$

Practice B Do every other part, and more if you need more practice.

1. On these, an order of conversion is specified. Write what must be placed in the blanks to make legal conversions, then solve.

   a. $? \frac{\text{miles}}{\text{hour}} = \frac{80.7\ \text{feet}}{\text{sec.}} \cdot \frac{\text{mile}}{\_\_\_\_\_\_} \cdot \frac{\_\_\_\_\_\_}{\text{min.}} \cdot \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} =$

   b. $? \frac{\text{meters}}{\text{s}} = \frac{250.\ \text{feet}}{\text{min.}} \cdot \frac{\text{min.}}{\_\_\_\_\_\_} \cdot \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} \cdot \frac{\_\_\_\_\_\_}{1\ \text{inch}} \cdot \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} =$

2. Add conversions in any order and solve.

   a. **?** $\frac{\text{km}}{\text{hour}} = \frac{1.17 \times 10^4 \text{ mm}}{\text{sec}}$

   b. **?** $\frac{\text{ng}}{\text{mL}} = \frac{47 \times 10^2 \text{ mg}}{\text{dm}^3}$

   c. **?** $\frac{\text{feet}}{\text{sec.}} = \frac{95 \text{ meters}}{\text{minute}}$

ANSWERS

Practice A

1. $? \frac{\text{g}}{\text{dL}} = 355 \frac{\text{g}}{\text{L}} \bullet \frac{\mathbf{10^{-1}} \text{ L}}{\mathbf{1} \text{ dL}} = 35.5 \frac{\text{g}}{\text{dL}}$

2. $? \frac{\text{meters}}{\text{s}} = \frac{4.2 \times 10^5 \text{ meters}}{\text{hour}} \bullet \frac{1 \text{ hour}}{60 \text{ min}} \bullet \frac{1 \text{ min}}{60 \text{ s}} = 1.2 \times 10^2 \frac{\text{meters}}{\text{s}}$

Practice B

1a. $? \frac{\text{miles}}{\text{hour}} = \frac{80.7 \text{ feet}}{\text{sec.}} \bullet \frac{1 \text{ mile}}{5{,}280 \text{ feet}} \bullet \frac{60 \text{ sec.}}{1 \text{ min.}} \bullet \frac{60 \text{ min.}}{1 \text{ hour}} = 55.0 \frac{\text{miles}}{\text{hour}}$

1b. $? \frac{\text{meters}}{\text{sec.}} = \frac{250. \text{ feet}}{\text{min.}} \bullet \frac{\mathbf{1} \text{ min.}}{\mathbf{60 \text{ sec.}}} \bullet \frac{\mathbf{12 \text{ inches}}}{\mathbf{1 \text{ foot}}} \bullet \frac{\mathbf{2.54 \text{ cm}}}{1 \text{ inch}} \bullet \frac{\mathbf{10^{-2} \text{ meter}}}{\mathbf{1 \text{ cm}}} = \mathbf{1.27} \frac{\mathbf{meters}}{\mathbf{s}}$

(<u>SF</u>: 250. due to the decimal has 3 *sf*, all other conversions are definitions, answer is rounded to 3 *sf*)

2a. $? \frac{\text{km}}{\text{hour}} = \frac{1.17 \times 10^4 \text{ mm}}{\textit{sec}} \bullet \frac{10^{-3} \text{ m}}{\mathbf{1 \text{ m}}\text{m}} \bullet \frac{\mathbf{1} \text{ km}}{\mathbf{10^3} \text{ m}} \bullet \frac{60 \textit{ sec}}{1 \text{ min}} \bullet \frac{60 \text{ min}}{1 \text{ hour}} = \mathbf{42.1} \frac{\mathbf{km}}{\mathbf{hr}}$

2b. $? \frac{\text{ng}}{\text{mL}} = \frac{47 \times 10^2 \textit{ mg}}{\mathbf{dm^3}} \bullet \frac{\mathbf{1 \text{ dm}^3}}{1 \text{ L}} \bullet \frac{10^{-3} \text{ L}}{1 \text{ mL}} \bullet \frac{\mathbf{10^{-3}} \text{ g}}{1 \textit{ mg}} \bullet \frac{1 \text{ ng}}{10^{-9} \text{ g}} = \mathbf{4.7 \times 10^6} \frac{\mathbf{ng}}{\mathbf{mL}}$

2c. Hint: an English/metric bridge conversion for distance units is needed. Head for the bridge: first convert the *given* metric distance unit to the metric distance unit used in your known bridge conversion.

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$? \frac{\text{feet}}{\text{sec.}} = \frac{95 \text{ meters}}{\text{min}} \bullet \frac{1 \text{ min}}{60 \text{ s}} \bullet \frac{1 \text{ cm}}{10^{-2} \text{ m}} \bullet \frac{\textit{1 inch}}{\textit{2.54 cm}} \bullet \frac{1 \text{ foot}}{12 \text{ in.}} = \mathbf{5.2} \frac{\mathbf{feet}}{\mathbf{s}}$

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Lesson 4F: The Atoms – Part 3

To continue to learn the most often encountered atoms, your assignment is:

- For the first 20 atoms**, plus** the first and last two *columns* in the periodic table, memorize the name, symbol, and the position of the atom. For each atom, given either its symbol or name, be able to write the other.
- Be able to fill in a *blank* table with those names and symbols.

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Periodic Table

1A	2A	3A	4A	5A	6A	7A	8A
1 H Hydrogen							2 He Helium
3 Li Lithium	4 Be Beryllium	5 B Boron	6 C Carbon	7 N Nitrogen	8 O Oxygen	9 F Fluorine	10 Ne Neon
11 Na Sodium	12 Mg Magnesium	13 Al Aluminum	14 Si Silicon	15 P Phosphorus	16 S Sulfur	17 Cl Chlorine	18 Ar Argon
19 K Potassium	20 Ca Calcium					Br Bromine	36 Kr Krypton
Rb Rubidium	Sr Strontium					I Iodine	54 Xe Xenon
Cs Cesium	Ba Barium					At Astatine	86 Rn Radon
Fr Francium	Ra Radium						

Lesson 4G: Review Quiz For Modules 1-4

Use a calculator and scratch paper, but no notes or tables. Write answers to calculations in proper significant figures. Except as noted, convert your answers to scientific notation.

To answer *multiple choice* questions, it is suggested that you

- Solve as if the question is *not* multiple choice,
- Then circle your answer among the choices provided.

Set a 20-minute limit, then check your answers after the *Summary* that follows.

1. (From Lesson 1C): $\dfrac{10^{23}}{(1.25 \times 10^{10})(4.0 \times 10^{-6})} =$

   a. 2.0×10^{18} **b.** 5.0×10^{18} **c.** 0.20×10^{19} **d.** 2.0×10^{20} **e.** 5.0×10^{-19}

2. (Lesson 1B): $(-60.0 \times 10^{-16}) - (-4.29 \times 10^{-14}) =$

   a. 4.8×10^{-16} **b.** 3.69×10^{-14} **c.** 3.7×10^{-14} **d.** 4.8×10^{-16} **e.** 4.89×10^{-14}

3. (Lesson 2A): 15 mL of liquid water has what mass in kg?

   a. 1.5×10^{-3} kg **b.** 15×10^{-4} kg **c.** 1.5×10^{-4} kg **d.** 1.5×10^{-4} kg **e.** 1.5×10^{-2} kg

4. (Lesson 2D): $5.00 \times 10^{-2}\ \dfrac{L^3 \bullet m}{s} \bullet 2.00\ m \bullet \dfrac{2.0\ s^3}{8.00 \times 10^{-5}\ L^2} \bullet$ (an exact 2) $=$

   a. $1.00 \times 10^{-4}\ m^2 \bullet s^2 \bullet L$ **b.** $5.00 \times 10^{3}\ m^2 \bullet s^2 \bullet L$ **c.** $5.0 \times 10^{3}\ m^2 \bullet s^2 \bullet L$

   d. $1.0 \times 10^{-3}\ m \bullet s^2 \bullet L$ **e.** $5.0 \times 10^{-3}\ m^2 \bullet s^2 \bullet L$

5. (Lesson 3B): State your answer in proper significant figures → but do not convert to scientific notation.

   $$\begin{array}{r} 1.008 \\ + \quad 238.00 \\ \underline{16.00} \end{array}$$

   a. 255.00 **b.** 255.0 **c.** 255.008 **d.** 255.1 **e.** 255.01

6. (Lesson 4D): If 1 kg = 2.20 lb., solve

   ? mg = 4.0×10^{-2} lb.

   a. 8.8×10^{-7} mg **b.** 8.8×10^{4} mg **c.** 1.8×10^{-7} mg **d.** 1.8×10^{4} mg **e.** 8.8×10^{1} mg

7. (4E): $? \dfrac{kg}{mL} = \dfrac{2.4 \times 10^{5}\ \mu g}{dm^3}$

   a. $2.4 \times 10^{-7}\ \dfrac{kg}{mL}$ **b.** $2.4 \times 10^{5}\ \dfrac{kg}{mL}$ **c.** $2.4 \times 10^{-10}\ \dfrac{kg}{mL}$ **d.** $2.4 \times 10^{-5}\ \dfrac{kg}{mL}$ **e.** $2.4 \times 10^{-4}\ \dfrac{kg}{mL}$

8. (Lesson 3D): For the following symbols, write the name of the atom.

   a. K = ___________________ b. S = _______________ c. Na = ______________

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SUMMARY: Conversion Factors

1. Conversion factors are fractions or ratios made from two entities that are equal or equivalent. Conversion factors have a value of one.
2. An *equality* can be written as a *conversion* or *fraction* or *ratio* that is equal to *one*.
3. When solving a problem, *first* write the *unit* WANTED, then an = sign.
4. Solving for single units, start conversion factors with

   ? unit WANTED = # and *UNIT given* • $\frac{\qquad}{\textit{UNIT given}}$

5. Finish each conversion factor with the answer unit or with a unit that takes you closer to the answer unit.
6. In making conversions, set up units to cancel, but add numbers that make legal conversions.
7. Chain your conversions so that the units cancel to get rid of the unit you've got and get to the unit you WANT.
8. When the unit on the right is the unit of the answer on the left, stop conversion factors. Complete the number math. Write the answer and its unit.
9. Units determine the placement of the numbers to get the right answer.
10. If you plan on a career in a science-related field, add these to your flashcard collection.

Front-side (with notch at top right): Back Side -- Answers

1 inch = ? cm	2.54 cm
1 kg = ? pounds	2.2 lb.
12 fluid oz. = ? mL	355 mL

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ANSWERS – Module 1-4 Review Quiz

Only *partial* solutions are provided below.

1. **a.** 2.0×10^{18} $1/5 \times 10^{23-10+6} = 0.20 \times 10^{19} = 2.0 \times 10^{18}$
2. **b**. 3.69×10^{-14} $(+\ 4.2\underline{9} \times 10^{-14}) - (0.600 \times 10^{-14})$ = net doubt in hundredths place
3. **e.** 1.5×10^{-2} kg 1.00 g H_2O = 1 mL H_2O ; 1.00 kg H_2O = 1 L H_2O
4. **c.** 5.0×10^{3} $m^2 \cdot s^2 \cdot L$ (2 *sf*)
5. **e.** 255.01 (Adding and subtracting, round to highest *place* with doubt.)
6. **d.** 1.8×10^{4} mg 7. **a.** 2.4×10^{-7} kg/mL L = dm^3

8a. K = Potassium b. S = Sulfur c. Na = Sodium

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